# The New Senior Secondary Curriculum for Sierra Leone 

## Subject Syllabus for Computer Mathematics <br> Subject stream: Mathematics and Numeracy



This subject syllabus is based on the National Curriculum Framework for Senior Secondary Education. It was prepared by national curriculum specialists and subject experts.

## Curriculum elements for Computer Mathematics - an applied subject

## Subject Description

Computer Mathematics aims to provide students with the requisite knowledge to gain an understanding of the mathematical concepts that underpin computers and how they store, process, communicate and transmit data. The students will be able to link theoretical mathematical concepts with its practical applications in computer programming, from logical and expressions and control structures for program flow, to matrices and functions for sophisticated data capture and analysis.

## Rationale for the Inclusion of Computer Mathematics in the Senior Secondary School Curriculum

Computer Mathematics is a unique subject in the Senior Secondary School curriculum. Not only does it go in greater depth in topics such as Base Number Arithmetic and Logic, which are usually only briefly covered in a typical Mathematics course, it introduces concepts such as Algorithms not typically found in a secondary school curriculum. Its inclusion broadens and deepens the scope of mathematics knowledge the student is exposed to and enriches the curriculum in turn. Students will develop knowledge and skills through studying Computer Mathematics which they can build on in their future studies and employment. Computer Mathematics can be studied along Applications of Computer Mathematics (Coding) and together they make an impressive package for acquiring $21^{\text {st }}$ century skills.

## General Learning Outcomes

At the end of the course, students will be able to:

- explain the concept of number systems including the Real Number System and Base Number System
- use data representation and number base arithmetic
- describe logic connectives and construct truth tables for logic gates
- understand and use algorithms through writing pseudocode and creating flowcharts
- use logical, arithmetic and relational expressions in pseudocode and flowcharts
- describe and use control structures in pseudocode and flowcharts
- solve linear equations, inequalities and formulas
- understand and use Set Theory
- understand and use probability
- describe character encoding systems
- solve simultaneous linear equations graphically, by substitution and elimination
- explain the fundamental principle of counting and use it to calculate probability
- understand permutations and combinations, and use them to calculate probability
- describe a function, its domain and range, inverse and composite functions
- understand and use matrices, including matrix operations and finding transpose and inverse of matrices
- use matrices to solve systems of linear equations


## Structure of the Syllabus Over the Three Year Senior Secondary Cycle

| SSS 1 | SSS 2 | SSS 3 |
| :---: | :---: | :---: |
| NUMBER SYSTEMS <br> - Number Systems Concepts <br> - The Real Number System <br> - Properties of Real Numbers <br> - Base Number Systems <br> - Conversions Between Number Bases <br> DATA REPRESENTATION AND BASE ARITHMETIC <br> - Computing Number Bases <br> - Units of Information <br> - Four operations on Binary Numbers <br> - Addition and Subtraction of Octal Numbers <br> - Addition and Subtraction of <br> - Hexadecimal Numbers | FURTHER BINARY ARITHMETIC <br> - Unsigned and Signed Binary Numbers <br> - Complements of Binary Numbers <br> - The Four Operations on Unsigned and Signed Binary Numbers <br> CHARACTER ENCODING SYSTEMS <br> - More on Hexadecimals <br> - Character Sets <br> - Character Encoding Systems <br> LOGIC II <br> - Tautologies and Contradictions <br> - Conditional Statements <br> - De Morgan's Laws <br> - Laws of Boolean Algebra <br> - XOR, NAND and NOR Logic Gates | MATRICES <br> - Basic Matrices Concepts <br> - Addition and Subtraction of Matrices <br> - Scalar Multiplication <br> - Matrix Multiplication <br> - Properties of Matrix Operations <br> - Determinant of Matrices <br> - Matrix Row Operations <br> - Inverse Matrices <br> SYSTEMS OF LINEAR EQUATIONS <br> - Solve Linear Equations in Two and Three Variables: Inverse Matrix Method <br> - Solve Linear Equations in Two and Three Variables: Gaussian Elimination Method <br> - Solve Linear Equations in $n$ Variables: Algorithm |
| LOGIC I <br> - Basic Logic Concepts <br> - Statements and Logical Connectives <br> - Truth Tables <br> - Boolean Logic <br> ALGORITHMS <br> - Basic Algorithm Concepts <br> - Pseudocode <br> - Flowcharts | SIMULTANEOUS LINEAR EQUATIONS <br> - Solve Linear Equations in Two Variables: Graphical Method <br> - Solve Linear Equations in Two Variables: Elimination Method <br> - Solve Linear Equations in Two Variables: Substitution Method <br> - Solve Linear Equations in Two Variables: Word Problems <br> - Solve Linear Equations in Three Variables: Graphical Method <br> - Solve Linear Equations in Three Variables: Elimination Method <br> - Solve Linear Equations in Three Variables: Word Problems | REVISION <br> - All Topics |

## LINEAR EQUATIONS, LINEAR INEQUALITIES AND FORMULAS

- Basic Algebra Concepts
- Linear Equations in One Variable
- Linear Inequalities in One Variable
- Graphical Representation of Linear Inequalities
- Formulas

SET THEORY I

- Basic Set Concepts
- Types of Sets
- Venn Diagrams
- Subsets and Proper Subsets
- Set Operations
- Properties of Set Operations
- De Morgan's Laws


## PROBABILITY I

- Basic Probability Concepts
- Experimental and Theoretical Probability
- Probability of Events
- Mutually Exclusive Events
- Independent Events
- De Morgan's Laws
- Conditional Probability


## SET THEORY II

- Cartesian Products of Sets
- Partition of Sets
- Power Sets


## PERMUTATIONS, COMBINATIONS AND

## PROBABILITY

- Fundamental Principles of Counting
- Multiplication Principle: Factorial Notation
- Permutations
- Combinations
- Probability of Events


## FUNCTIONS

- Mappings, Relations and Functions
- Using Function Notation
- Types of Functions
- Representing Functions
- Domain and Range of Functions
- Inverse Functions
- Composite Functions


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| Term 2 | LOGIC I <br> - Basic Logic Concepts <br> - Statements and Logical Connectives <br> - Truth Tables <br> - Boolean Logic <br> ALGORITHMS <br> - Basic Algorithm Concepts <br> - Pseudocode <br> - Flowcharts <br> LINEAR EQUATIONS, LINEAR INEQUALITIES AND FORMULAS <br> - Basic Algebra Concepts <br> - Linear Equations in One Variable | SIMULTANEOUS LINEAR EQUATIONS <br> - Solve Linear Equations in Two Variables: Graphical Method <br> - Solve Linear Equations in Two Variables: Elimination Method <br> - Solve Linear Equations in Two Variables: Substitution Method <br> - Solve Linear Equations in Two Variables: Word Problems <br> - Solve Linear Equations in Three Variables: Graphical Method <br> - Solve Linear Equations in Three Variables: Elimination Method <br> - Solve Linear Equations in Three Variables: Word Problems | REVISION <br> - All Topics |

- Linear Inequalities in One Variable
- Graphical Representation of Linear Inequalities
- Formulas

Term 3
SET THEORY I

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- Multiplication Principle: Factorial Notation
- Permutations
- Combinations
- Probability of Events


## FUNCTIONS

- Mappings, Relations and Functions
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## Teaching Syllabus



$$
\text { and }(a \cdot b) \cdot c=a \cdot(b \cdot c))
$$

- identity (or neutral) elements ( $a+0=a$ and $a \cdot 1=a)$
- inverse elements

$$
\begin{aligned}
& \left(a-a=a \times \frac{1}{a}=1\right. \\
& \text { for } a \neq 0)
\end{aligned}
$$

Show they understand that subtraction and division are neither associative nor commutative

Show they understand and can use the positional number systems of bases: decimal, binary, octal, hexadecimal, and their notations

Convert between decimal,
binary, octal and
hexadecimal systems

Demonstrate and guide students to use the inequality symbols, <, $\leq,>, \geq$ to compare the numbers

Demonstrate each of the properties using a combination of real numbers, i.e., integers, rational and irrational numbers, e.g., for the commutative property for multiplication, show that:
$7 \times 8=8 \times 7$
Guide students to test some of the properties themselves,
e.g., How can we show that
$\frac{12}{\sqrt{14}} \times \frac{1}{\frac{12}{\sqrt{14}}}=1$
For commutativity, guide students to show on a number line why they get the same answer for addition and multiplication of real numbers

Guide students to compare what they get when they do the calculation, $6-2$ and $2-6$
number in the
corresponding base on the other)

Place-value charts for decimal, binary, octal, hexadecimal number systems
and $(a \cdot b) \cdot c=a \cdot(b \cdot c)$
Answer questions such as:

- "Why is subtraction not commutative?"
- "Why is multiplication associative?"

Show on a number line that real numbers are commutative and associative under:

$$
\begin{aligned}
& 9+4=4+9 \\
& \text { and } \\
& 3 \times 9=9 \times 3
\end{aligned}
$$

Answer standard questions on properties of real numbers

Answer questions such as:
"What are the place values for base 2 (8 / 16)"

Read and write a decimal, binary, octal or hexadecima number using the correct base notation

Create a table (or poster) of the first 16 decimal, binary, octal and hexadecimal numbers

Do they get the same answer? Can they show the calculation on a number line to explain why they get different answers?

Guide students to do the same for division and explain their result

Demonstrate and guide student to show the positional (place-value) nature of these systems using place-value charts with the positions being represented by the powers of the base

Guide students to use the base notation, e.g., 568 (for an octal number, or \# or 0x prefix for
hexadecimal
Guide students to use dot cards to show how counting is done in decimal (base 10), binary (base 2), octal (base 8) and hexadecimal (base 16)

Discuss correct counting, e.g., " 56 " in octal is not pronounced "fifty-six", but "five six"; fifty six is a

Work in pairs to match cards with the same decimal,
binary, octal and
hexadecimal number
Work independently or in pairs to use the appropriate powers to convert between different base systems
decimal number (five tens and six ones)

Demonstrate using the appropriate powers of the base how to convert:

- a number in decimal to binary, octal or
hexadecimal, and vice versa
- a number in binary to octal and vice versa
- a number in binary to hexadecimal and vice versa
Explain what bits and bytes are in terms of binary numbers

Guide students to use the information sheets to show how binary numbers are used in computer systems. For example, a bit is a binary digit, 0 (for off) or 1 (for on). It is the unit of information computers use to store data (i.e., numbers, text, sound, images, etc.)

Demonstrate and guide students to show how octal and hexadecimal numbers are used in writing code as they have fewer number of digits

## Textbook

Information sheets on the use of non-decimal number systems in computers

Activity sheets

Students are able to:

Explain to a friend who was absent why binary, octal and hexadecimal numbers are used in computer programming

Can use conversions of decimal, binary, octal and hexadecimal numbers to explain data representation in computers

Convert between units of information (e.g. bytes, kilobytes, megabytes, etc.)

Use the place value of binary numbers to add and subtract two binary integer numbers

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which are easier to
manipulate than binary
numbers. For example:

\[\)| $89_{10}$ | $=1011001_{2}$ |
| ---: | :--- |
|  | $=01011001_{2}$ |
|  | $=001011$ |
|  | $=131_{8}$ |

\]

Also, $\quad$| $89_{10}$ | $=0101$ |
| ---: | :--- |
|  | $=5916$ |

Use the place value of octal and hexadecimal numbers to add and subtract integer numbers

Ask students how they would translate these numbers into bits

Introduce the terms: byte (a group of 8 bits), nibble for half a byte (4 bits),
kilobyte (KB), megabyte (MB), gigabyte (GB) and tetrabyte (TB)

Remind students that computer storage (hard drive and RAM) is measured in (tens of) thousands of bytes

Demonstrate how to complete the addition facts table for binary integer numbers, i.e.
$+01111$
+0 1 111
$0 \quad 0 \quad 1$
$+\quad 101$
+1100

Use the addition table to guide students in adding two binary integer numbers, of no more than 8 bits per number, as
shown above
Demonstrate and guide students to subtract two binary integer numbers of no more than 8 bits per number

Briefly explain overflow errors in calculations

For each base, guide students to complete the addition facts in tables
e.g., Octal addition

+ 01234567
$\begin{array}{lllllllll}0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7\end{array}$
$\begin{array}{lllllllll}1 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 10\end{array}$
22345671011
$\begin{array}{lllllll}3 & 3 & 4 & 5 & 6 & 7 & 101112\end{array}$
$\begin{array}{llllll}4 & 4 & 5 & 6 & 7 & 1011\end{array} 1213$
$5 \begin{array}{lllll}5 & 6 & 7 & 1011121314\end{array}$
667101112131415
7710111213141516
Ask questions such as "Is there a pattern to the results obtained from adding two numbers together which can help us to add two octal (hexadecimal) numbers?"

|  |  | Use the addition facts to demonstrate how $1: 1$ addition is performed on + 376 octal integer 1035 numbers. <br> Use similar reasoning to add hexadecimal integer numbers and to subtract in each of the bases. Limit the numbers to no more than 8 bits equivalent per number |  |  |
| :---: | :---: | :---: | :---: | :---: |
| YEAR 1/TERM 2 |  |  |  |  |
| LOGIC I |  | Ask students what they understand by the word | Textbook Dictionary | Students are able to: |
| Basic Logic Concepts <br> Statements and Logical Connectives | Understand the concept of mathematical logic and its applications in computer programming | 'logic'. Allow them to use their dictionaries and share the meaning found with the class. | Information sheets on mathematical logic Internet <br> Activity sheets Logic puzzles and games | research and write a brief report on the history of mathematical logic and its applications in computer programming |
| Boolean Logic | Identify open and closed statements | Guide students to use the information sheets to explain what logic is (i.e., |  | Work in pairs to identify open and closed statements and |
|  | Assign a truth value to a statement <br> Write the symbolic form of simple statements | a science that studies the principles of correct reasoning). Ask: "Does it match the dictionary meaning?" Discuss why/why not |  | give reasons for choice <br> Assign a truth value to a statement by identifying them as true or false |
|  | Classify compound statements as a negation, conjunction, disjunction, conditional or bi-conditional | Guide students to research and discuss why logic is important in computer programming |  | Work in pairs to form statements for their partner to identify as true or false and give reasons for choice |
|  | Use logical connectives to | Discuss open and closed |  | Write simple statements using the form: |

write compound statements
Write a compound statement given in words in symbolic form, and vice versa

Use truth tables to analyse the truth values of compound statements

Determine if statements are logically equivalent.

Show they understand the concept of Boolean logic and its applications in computer programming

Show they understand and can use operators in expressions to control the flow in computer programming:

- Boolean (or logical) operators
- relational operators
- < (less than)
- <= (less than or equal to)
- > (greater than)
- >= (greater than or equal to)
- = (equal to)
- $\neq$ (not equal to)
- arithmetic operators
statements using everyday examples such as:
Mangoes are fruits
Mangoes are the best
fruits
Guide students to identify simple open and closed statements and discuss reasons for choices

Demonstrate and guide students to use simple statements which can
only be answered by 'True' or 'False' to explain the concept of mathematical logic, e.g
4 is an even number
Sierra Leone is in West
Africa
All prime numbers are odd
$1+2=3$
Which of these are true
$(\mathrm{T})$ / false ( F ) ?
How do they know the statement is true/false? Can they justify their answer?

Show how letters can be used to denote simple statements, e.g., let $p$ represent the simple statement: "Mariama is a farmer".

Let $p$ : grass is green
Let $q$ : every even number is divisible by 4

Work independently or in pairs to identify compound statements

Use letters and the logical connectives to translate compound statements written in words to symbols and vice versa

Write simple statements in words for a partner to form compound statements in symbols to represent:

$$
\begin{aligned}
& (p \square q) \square r \text { and } \\
& p \square(q \square r)
\end{aligned}
$$

Discuss in their pairs whether the two statements mean the same thing. Discuss as a class.

Work independently or in pairs to complete logic puzzles and other logic games

Use the truth tables for $\sim p, p$
$\square \boldsymbol{q}, p \square q$ to construct new
truth tables, e.g. for $p \square \sim q$
$p \square p \square q$, etc.

Show that this statement can be written as:
Let $p$ : Mariama is a
farmer
Demonstrate and guide students to classify compound statement as a negation or a combination of simple statements using logical connectives:

- negation (not, ~)
- conjunction (and, $\square$ )
- disjunction (or, $\square$ )
- conditional (if then, $\square$ )
- bi-conditional (if and only if, ■)

Demonstrate using plenty of examples of increasing complexity to illustrate each of the compound statements.
e.g. (of negation)
the statement:
"Musa is at home" has as its negation "Musa is not at home"
Show how this can be written as:
Let $p$ : Musa is at home
$\sim p$ : Musa is not at
home
$\sim p$ is read as "not $p$ "
Demonstrate that
negations of true
statements is always

Answer questions such as:

- "Under what conditions will a given compound statement be true?"

Devise a strategy to help a friend systematically construct truth tables and thus determine its truth value

Work independently or in pairs to complete logic puzzles and other logic games

Draw truth tables to determine if statements are logically equivalent

Explain Boolean logic and how it is used in computer programming

Construct logic gates and truth tables for NOT, AND and OR from both everyday life and mathematics

Write expressions using both Boolean and relational operators

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## false and vice versa

Guide students to
interpret and write compound statements including negations in words and symbolic form

Introduce truth tables using an example in words for negation:
Let $p$ : Musa is at home
$\sim p$ : Musa is not at home

| $p$ | $\sim p$ |
| :--- | :--- |
| T | F |
| F | T |

If $p$ is true, $\sim p$ is false. If $p$ is false, $\sim p$ is true

Explain that a truth table is used to determine whether a compound
statement is true or false
Demonstrate further
examples using
mathematical statements,
e.g.

Let $p$ : 2 is an even number
$\sim p: 2$ is not an even
number

Guide the students to work in pairs. They use statements written in words to construct truth tables for negation, conjunction, disjunctive and the conditional statements

Demonstrate using examples the conditions under which two statements are logically equivalent. Guide students to draw truth tables and understand this happens if and only if their truth tables are the
same
Introduce and discuss
Boolean logic as the logic used to control the flow of computer programs

Guide students to
construct truth tables to represent the Boolean
logic gates:
NOT ( ${ }^{-}$

- AND (+),

OR (•)
where the binary digits 1 and 0 indicate True (on)
and False (off)
respectively
Guide students to write
the Boolean expression

## for each gate

## Discuss and guide

students to show how the
Boolean operators are used giving examples from everyday life such as with the statements:
A: I will take a keke if it is
very hot
$B$ : I am tired
Demonstrate and guide students to use the
typical relational
operators in conjunction with boolean operators, e.g.
if Age < 16 OR Age > 65 then you pay a reduced price
will result in either true or false depending on the age of the person

Demonstrate and guide students to use the order of operations for Boolean operators, I.e.
(Brackets) - NOT - AND - OR

## ALGORITHMS

Basic Algorithm Concepts

Pseudocode

Students will be able to:
Define and state the characteristics of algorithms

Outline functions of algorithms

Guide students to work in pairs to put a set of sequencing cards for a task, (e.g., making tea or coffee), in order. Alternatively ask students to think and write down

Textbook Activity sheets Sequencing cards Poster of flowchart names, symbols and uses

Students are able to
Explain what an algorithm is, its characteristics and functions

Write algorithms to solve

## Flowcharts

Explain and use the components of pseudocode
to write simple algorithms for solving given problems

Define and state the characteristics of flowcharts

Classify flowchart symbols and their uses

Use control structures to translate more complex algorithms to pseudocode and flowchart
sequence:

- a linear execution of statements
- selection (conditionals):
- if/then
if/then/else
- case
- Boolean logic
repetition (loops):
- while
- for
- do/while
- repeat/until

Draw flowcharts for solving given problems
the activities they need to do to complete the task

Ask a few students to put their solutions on the board. Compare the solutions, ask questions on students' thinking (see Assessment) and vote on the most efficient (or quickest) solutions

Explain what an algorithm is using the task students just completed

Provide explanations of characteristics and functions of algorithms

Show examples of simple algorithms written in pseudocode. Use them to explain the basic pseudocode (and coding) components:

- variable - unknown quantity with a name, a data type, and a value.
- assignment - give a value to a variable
- transfer - read an input, write an output
- control - specifies which is the next step to be executed
- a simple everyday problem, e.g., buying a mobile phone
- a simple mathematical problems, e.g. finding the area of a rectangle given the lengths of its sides

Answer questions such as:

- "What made you decide to do it that way?"
- "What can you do to make your algorithm more efficient?"
- "What is the same / different between your solution and one on the board?"

Use the components of pseudocode to write simple algorithms for given problems Problems include:

- adding three numbers
- finding the average of two or three numbers
- calculating perimeter, areas and volumes of shapes, etc.


## Answer questions such as:

- "What is a flowchart?"

Explain to an absent classmate the characteristics,

Guide students to write algorithms using pseudocode, e.g., add two numbers

Show an example of a simple flowchart (e.g., for adding two numbers) and use it to define and state the characteristics of flowcharts

Demonstrate using a prepared poster the names and pictures of flowchart symbols and their uses in drawing flowcharts

Demonstrate and guide students to use pseudocode and flowcharts to show how more complex algorithms and programs are written using the three control structures

Discuss sequential control as the default means by which a program is executed as used in the basic algorithms to date

Explain how selection control is used to execute one or more statements if a given condition is met
names, symbols and uses of flowcharts

Use the symbols of flowcharts to write simple algorithms for the given problems above

Trace the logic of given pseudocode and flowcharts which show more complex problems using control structures

Sse pseudocode and flowcharts to write more complex algorithms to solve a given problem.
Problems include:

- find the largest / smallest number among three numbers
- generate the 5 times tables from $1 x$ to $12 x$
- output the count of all even numbers between a user defined range of numbers
- check whether a number is prime or not (composite)
- write error message when input number is not 5 or 6
- etc.

Compare their algorithms with other students and improve their own

Guide students to research the internet to find out how iteration control works \{repeats a statement a certain
number of times, or while a condition is fulfilled)

Guide students to work in pairs to write pseudocode and draw flowcharts for
more complex
mathematical
calculations, e.g., finding the larger /smaller of two numbers

Compare and discuss students' pseudocodes for efficiency
Review the basic algebra Textbooks concepts using examples Activity sheets
and exercises for
students to complete.
Include algebraic
expressions with
fractions.
Demonstrate collecting
like terms using a
combination of like terms,
e.g., $x^{2} y x y^{2}$, etc. Include negative coefficients

Demonstrate and guide
students to solve
equations of the type

## Students are able to

Recall and answer questions on basic algebraic concepts

Solve linear equations in one variable

Check solutions to linear equations

Solve word problems involving linear equations in one variable

Solve linear inequalities in one variable and show the

| Solve word problems involving linear equations in one variable | $\mathrm{ax}+\mathrm{b}=\mathrm{c}$ <br> using the addition and multiplication principles |
| :---: | :---: |
| Solve linear inequalities in one variable | Demonstrate other types, of linear equations e.g., with the variable on both |
| Represent the solution sets of linear inequalities graphically | sides of the equation, e.g. $a x+b=c x+d$ |
| Solve word problems involving linear inequalities in one variable | Use examples which will require expanding brackets, collecting like terms and fractions |
| Substitute values into formulas <br> Change the subject of | Guide students to check their solutions by substituting into the original equation |
|  | Guide students to solve word problems involving linear equations in one variable |
|  | Demonstrate and guide students to solve a variety of linear inequalities, e.g. $0 \leq a x+b \leq c$ |
|  | Guide students to represent the solution on a number line Guide students to solve word problems involving linear inequalities in one variable |


description method, roster form, and set-builder notation

Use Venn diagrams to represent sets

Classify sets as either finite or infinite

Find the cardinality of a set
Explain the conditions under which two sets are equal, equivalent, both or neither

Identify disjoint sets
Identify the unit set
Identify, and use the notation for, the empty (null) set
dentify, and use the notation for, the universa set

Identify, and use the notation for, the complement of a set

Use the notation for subsets of a set

Find all subsets and proper subsets of a set

Find the union of two sets
students to understand
the concept of well-
defined sets using examples and nonexamples

Demonstrate and guide students to name sets using capital letters, and elements using the $\in$ symbol, i.e.: $p \in A$, which is read as:
$p$ is an element of $A$
Guide students to use the proper notation for an element which does not belong to a set, i.e.: $p \notin A$ " $p$ is not an element of $A$ "

Guide students to give everyday examples of sets and list elements of the set.

Demonstrate how to write sets using:

- description of their common attributes, e.g., the set of all natural numbers
- roster form using curly brackets and ellipsis where appropriate, e.g.

$$
N=\{1,2,3, \ldots\}
$$

- set-builder notation, e.g.
with same initia
Write a given set in the other two ways

Write sets in one of the forms from a table of data or graph, e.g. given a table of population of districts in Sierra Leone, they can write sets of districts with population more (or less) than 100,000, etc.

Work independently or in pairs to sort a variety of given sets into finite, infinite, unit and empty sets

Sort pairs of sets into equal, equivalent, both or neither

Answer questions such as:

- "How many ways can you find to describe the set ...?"
- "What elements are in the complement of the set ....? Draw a Venn diagram to represent the set


## Describe five possible

 universal sets of which Sierra Leone is one of the elements, e.g., set of countries in WestAfrica, set of diamond exporting countries, set of countries who have hosted the Dakar rally, etc.

Find the intersection of two sets

Find the difference of two sets

Show they understand and can use properties of set operations:

- commutative
- associative
- distributive
- identity
- dominative
- complement
- De Morgan's Law
$N=\{x: x$ is a natural number $\}$ read as: "set $N$ is the set of all elements x such that x is a natural number"

Provide examples of different sets. Guide students to work in groups to investigate each of the statements below by listing elements and drawing Venn diagrams of different types of sets:

- finite set (a set with limited number of elements/members)
- infinite set (a set with unlimited number of elements/members)
- cardinality of a set (the (number of elements, $n$, in a set $A, n(A))$
- equal sets (two sets,
$A, B$, containing exactly the same elements, i.e. $A=B$ )
- equivalent sets (two sets $A, B$ containing the same number of elements, $n(A)=$ $n(B)$ )
- disjoint sets (two sets $A, B$ containing no

Answer questions such as:

- "What sets can you make for a different element
such as ...?"
- "Name another element and the different sets in which it can be put"
- "Name all the subsets of the set ..."
- "Which of the subsets are also proper subsets of the set ...?"

Answer standard questions using set notation and Venn diagrams on subsets and proper subsets of a given set

Answer standard questions using set notation and Venn diagrams on union,
intersection and difference, of up to three sets

Answer standard questions using set notation and Venn diagrams on properties of up to three sets
elements in common)

- unit set (a set, $A$, with a single member,

$$
n(A)=1)
$$

- empty (null, Ø) set (a set with no elements or members)
- universal set, $U$ or $\varepsilon$ (a set containing all the elements in a given context)
- complement of a set (a set containing all the elements not in a particular set, $\mathrm{A}^{\mathrm{c}}$ or A')

Demonstrate and guide students to describe and draw Venn diagrams for:

- subset (a set $A$ containing all the elements of another set $B$, i.e. $A \square B$; sometimes the two sets can also be equal). Show that in a set with $n$ elements the number of subsets is $2^{n}$
- proper subset (a set containing all the elements of another set, but is not equal to that set, i.e. $A \square B, A$ $\square B)$. Show that in a set with $n$ elements the number of proper

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## subsets is $2^{n}-1$

Discuss the following (or similar) example with the students:
if $A=\{2,4,6\}$ then $B=\{2,6\}$ is a proper subset of $A$.
The set $\mathrm{C}=\{2,4,6\}$ is a
subset of $A$, but it is not a proper subset of $A$ since
$\mathrm{C}=\mathrm{A}$.
The set $D=\{2,5\}$ is not even a subset of $A$, since
5 is not an element of $A$.
Demonstrate and guide students to use set notation and Venn diagrams to find:

- union of two sets, $A \square B$
- intersection of two sets, $A \square B$
- difference of two sets,

$$
A \mid B(\text { or } \mathrm{A}-\mathrm{B})
$$

Demonstrate and guide
students to use set
notation and Venn
diagrams to show that
given sets $A, B, C$ :
commutativity
$A \square B=B \square A$
$A \square B=B \square A$
associativity
$(A \square B) \square C=A \square(B \square$
C)
$(A \square B) \square C=A \square(B \square$

|  |  | C) <br> - distributivity <br> $A \square(B \square C)$ $=(A$ $\text { B) } \square(A$ <br> $A \square$ ( $=(A$ $\text { B) } \square$ $(A$ <br> - identity <br> $A \square \varnothing=A, A \square U=A$ - dominativity <br> A $U=U, A \square \varnothing=\varnothing$ idempotent <br> A $A=A, A \square A=A$ complement <br> $A \square A^{c}=U, A \square A^{c}=\varnothing$ De Morgan's Law <br> ( $A$ $\square$ $B)^{c}=A^{c}$ $\square$ $B^{c}$ <br> (A $B)^{c}=A^{c} \square$ <br> Guide students to solve standard problems using these concepts <br> Demonstrate and guide students to look at other laws if time permits (e.g. double complement, absorption, etc.) |  |  |
| :---: | :---: | :---: | :---: | :---: |
| PROBABILITY I <br> Basic Probability Concepts | Students will be able to: <br> Demonstrate that they understand the language of probability | Use practical examples, e.g., rolling a single die to explain the meaning of the words experiment, event, outcome, sample | Textbooks Activity sheets Computers Internet Dice | Students are able to: <br> Explain the terms used in probability, e.g., experiment, event, outcome, etc. |
| Experimental and Theoretical Probability <br> Probability of Events <br> Mutually Exclusive | Explain the use of probability in computer programming <br> Find the experimental | space, fair, bias Work independently or with a partner to classify given events according to whether they are Certain, Likely, Unlikely or | Playing cards Coins Spinners | Explain why probability is important to computer programming <br> Use dice, playing cards, |



## in question

Demonstrate and guide students to write the sample space for equally likely events and calculate their theoretical probabilities using the formula:
$P(E)=\frac{\text { number of outcomes of a a event }}{\text { total number of outcomes }}$
Guide students to use the results from their experiments and the theoretical sample spaces to differentiate between experimental and theoretical
probabilities
Discuss what happens with the experimental probability when a sufficiently large number of experiments is performed

Maybe move this up + the probability line Demonstrate how individual probabilities of events are between 0 and 1 , and all the probabilities add up to 1

Guide students to find

Calculate and verify that the probabilities of an experiment add up to 1

Show probabilities of events on a number line

Use the addition law and Venn diagrams to calculate and show the probability of two events happening
use the addition law for mutually exclusive events, and Venn diagrams to calculate and show the probabilities of mutually exclusive events

Use tree diagrams to find the probability of events

Use the product law to calculate the probabilities of independent events

Use Venn diagrams to illustrate De Morgan's laws

Solve problems using the principles of De Morgan's laws. Illustrate answers using Venn diagrams

Answer standard questions on conditional probability

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probability of event $B$, given probability of event
$A$ as:
$P(B)=1-P(A)$
Draw a probability line on the board and explain its features. Demonstrate and guide students to show the probabilities of simple events on the line

Demonstrate using Venn diagrams the probability of two events happening


Show how this can be written as:
$P(A)+P(B)-P(A \square B)$
Guide students to use the addition law to find probabilities and show
their results on Venn
diagrams
Demonstrate and illustrate using Venn diagrams to show mutually exclusive events as events which do not occur at the same time

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Guide students to state and use the addition rule to find the probability of two mutually exclusive events $A$ or $B$ occurring, i.e., from the diagram: $(A \square B)=0 \square P(A \square B)=$ 0
$\square P(A$ or $B)=P(A)+P(B)$
Demonstrate using practical examples, e.g., the probability of getting a head and a tail at the same time when tossing a coin

Introduce tree diagrams and use it to find simple probabilities

Demonstrate and guide students to use tree diagrams to show two independent events defined as the probability of one event occurring having no effect on the probability of the other event occurring.

Guide students to state and use the product rule

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to find the probabilities of two independent events
occurring, i.e.
$P(A$ and $B)=P(A) \times P(B)$
Demonstrate using
practical examples, e.g., the probability of getting a head on a coin toss and drawing a 3 of hearts from a pack of playing cards

Discuss De Morgan's
Laws (met previously in
Sets) as it relates to
probability, i.e.
$P(A \square B)^{c}=P\left(A^{c} \square B^{c}\right)$
$P(A \square B)^{c}=P\left(A^{c} \square B^{c}\right)$
Demonstrate practical examples using Venn diagrams.

Guide students to use the laws and Venn diagrams to solve problems

Use Venn diagrams to illustrate and guide students to solve problems

Guide students to calculate simple conditional probabilities i.e., Probability of event A, given that B had
occurred is defined by:

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$$
P(A \mid B)=\frac{P(A \cap B)}{P(B)}
$$

Show using practical
examples that conditional
probability is the
probability of event $A$
occurring, given that
event $B$ occurs.
Note if $A$ and $B$ are independent then
$P(A \mid B)=P(A)$
Guide students to use the
formulas to calculate conditional probability of an event, e.g., given that
a black card is drawn
from a pack of cards, the probability of it being a
seven, i.e.,
$P$ (seven|black)

YEAR 2/TERM 1

## FURTHER BINARY

## ARITHMETIC

Unsigned and Signed
Binary Numbers
Complements of Binary
Numbers
The Four Operations on Unsigned and Signed Binary Numbers

Students will be able to:

Show that they know the difference between unsigned and signed binary numbers

Show they understand and can use complements of binary numbers

Recall and extend binary addition and subtraction

Perform multiplication and division of unsigned binary integer numbers

Perform multiplication and division of signed binary integer numbers

Review unsigned binary numbers

Demonstrate and guide students to show signed numbers represented through

- sign and magnitude notation (SM) using the most significant bit (MSB)
- one's complement
- two's complement

Review how to add and subtract up to three unsigned binary numbers of no more than 8 bits per number

Demonstrate and guide students to add and subtract unsigned and signed binary numbers using one's and two's complements

Demonstrate and guide students to apply a binary shift to multiply and divide two unsigned binary integer numbers of no more than 8 bits per number

Demonstrate and guide students to use two's

## Textbooks Activity sheets

ferentiate between unsigned and signed binary numbers

Explain and convert numbers using different
representations
Add and subtract unsigned and signed binary integer numbers using a variety of methods

Multiply and divide unsigned and signed binary integer number using a variety of methods including binary shift and two's complement
complement to multiply and divide unsigned and signed binary integer numbers of no more than 8 bits per number
Recall that a number such as $89_{10}$ can be written as a 7 -bit number, two groups of 4-bit numbers (nibble), and as $59_{16}$ as shown below:
$89_{10}=1011001_{2}$
$=01011001_{2}$
$=59_{16}$
Demonstrate and guide students to write each hexadecimal (hex) from 0 to 25510 in groups of 4 bits

Explain a character set as a defined list of characters (e.g., alphanumeric),
recognised by the
computer with each
character (e.g. the letter
q) represented by a
number
Demonstrate and guide
students to use a
character encoding table
to:

- convert characters to character codes

Textbooks
Activity sheets
Character encoding table
for ASCII, Unicode

Students are able to:
Write any decimal number
from 0 to $255_{10}$ in
hexadecima
Identify the types of characters found in a character set

Convert characters to character codes and vice versa

Write names of people and places in ASCII and Unicode

Explain the benefits and limitations of ASCII and Unicode

|  |  | - convert character codes to characters <br> Guide students to investigate the benefits and limitations of ASCII and Unicode |  |
| :---: | :---: | :---: | :---: |
| LOGIC II <br> Tautologies and contradictions | Students will be able to: <br> Identify tautologies and contradictions | Discuss using examples; tautology, a statement which is always true; and contradiction, a statement which is always false. | Textbooks Activity sheets |
| Conditional Statements <br> De Morgan's Laws | Recall and extend conditional statements to: | Guide students to identify these two types of |  |
| Laws of Boolean Algebra | - identify the hypothesis and conclusion of a conditional statement <br> - convert statements to | statements from everyday and mathematical statements |  |
| XOR, NAND and NOR Logic Gates | the standard ("if ... then") form write the converse, inverse, and contrapositive of a conditional statement describe a counterexample for a conditional statement write the negation of a conditional statement. <br> State and use De Morgan's laws to determine if statements are logically equivalent. | Use everyday and mathematical statements to demonstrate and guide students to extend their understanding of conditional statements <br> Students use given statements and identify, write, convert and otherwise describe statements as required |  |

convert character

Guide students to investigate the benefits and Unicode
Discuss using examples which is a ways true; and is always true; and contradiction, a statemen nich is always false.

Guide students to identify wo types of tements from everyday and mathematical

Use everyday and mathematical statements to demonstrate and guid students to extend their understanding of

Students use given statements and identify, convert and therwise describe statements as required laws to determine if equivalent.

## Textbooks

Activity sheets
Students are able to:

Identify given statements as being a tautology or contradiction

Use given statements and identify, write, convert and otherwise describe statements as required

Use De Morgan's Laws to write statements that are equivalent to a given statement

Draw truth tables for two statements to verify if they are logically equivalent

Use the laws of Boolean algebra to solve standard logic questions including constructing truth tables

Interpret the results of simple truth tables

Solve problems on combinations of logic gates

State and apply the laws of Boolean algebra

- commutative
- associative
- distributive

Recall and extend use of the Boolean operators to include:

- XOR
- NAND
- NOR

Describe more complex situations using combinations of logic gates

Recall equivalent
statements as statements
if and only if their truth
tables are the same
Discuss De Morgan's
laws, (previously met in Sets in SSS1):

- "not (A and B)" is
logically equivalent to "not A or not B"
$\sim(p \square q) \equiv \sim p \square \sim q$
- "not (A or B)" is logically equivalent to "not A and not B"
$\sim(p \square q) \equiv \sim p \square \sim q$
Use examples to show the laws of Boolean algebra, including: commutativity
$A \cdot B=B \cdot A, A+B=B+$ A
- associativity
(A•B).C $=A \cdot(B \cdot C)$
$(A+B)+C=A+(B+C)$
- distributivity
$A \cdot(B+C)=A \cdot B+A \cdot C$
$A+(B \cdot C)=(A+B) \cdot(A+$
C)

These rules only apply to
AND and OR
Guide students to construct truth tables for these laws

Review the NOT, AND and OR including
constructing truth tables for the logic gates

Guide students to construct truth tables for the logic gates:

- XOR
- NAND
- NOR

Guide students to write the Boolean expression for each gate

Demonstrate and guide students to draw truth tables and write Boolean expressions for logic gates of increasing complexity

Demonstrate and guide students to use the graphical method to solve two linear equation in two variables of the form:
$a x+b y=c$
Use sets of equations of increasing complexity,
e.g.:

- $5 x+2 y \square 10$
$x \quad \square 3$
- $5 x+2 y \quad \square 10$
$y \quad \square 4$
- $5 x+2 y \quad 10$


## SIMULTANEOUS

## LINEAR EQUATIONS

Solve Linear Equations in
Two Variables: Graphical Method

Solve Linear Equations in Two Variables: Elimination Method

Solve Linear Equations in
Two Variables:
Substitution Method
Solve Linear Equations in Two Variables: Word

Students are able to

Solve linear equations in two variables using the graphical method

Solve linear equations in two variables using elimination method

Solve linear equations in two variables using the substitution method

Solve word problems involving linear equations in two variables

Textbooks
Activity she Activity sheets

## 




Students are able to:
Solve linear equations in two variables using the graphical method

Solve linear equations in two variables using elimination method

Solve linear equations in two variables using substitution method

Solve word problems involving linear equations in two variables using any

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| Problems |  | $2 x-\quad y \quad \square 4$ | method |
| :---: | :---: | :---: | :---: |
| Solve Linear Equations in Three Variables: Graphical Method | Solve linear equations in three variables using the graphical method <br> Solve linear equations in | Use the elimination and substitution methods to solve the same problems above | Solve linear equations in three variables using the graphical method |
| Solve Linear Equations in Three Variables: Elimination Method | three variables using the elimination method <br> Determine whether a given | Guide students to use any method to solve word problems involving two | Solve linear equations in three variables using the elimination method |
| Solve Linear Equations in Three Variables: Word Problems | ordered triple is a solution to a three=by=three system of equations | linear equations involving two variables | Substitute given triples in given three-by-three systems to verify if they are solutions |
|  | Solve word problems involving linear equations in three variables | Demonstrate and guide students to use the graphical method to solve three linear equation in three variables (known as three-by-three, $3 \times 3$, system) of the form: $a x+b y+c z=d$ | Verify own solutions to check if they have a valid solution <br> Solve word problems involving linear equations in three variables using the elimination method |
|  |  | Demonstrate how to solve three linear equations in three variables |  |
|  |  | Demonstrate and guide students to substitute given triple into each equation in turn to verify whether or not it satisfies the equation. All three equations must be satisfied for the triple to be a solution |  |
|  |  | Demonstrate how to set up and solve the three |  |



## Textbooks Activity sheets <br> Students are able to: <br> Describe the Cartesian product of sets <br> Write the Cartesian product of up to 3 sets <br> Answer questions such as:

- "How do you know you have found all the sets?
- "Is there a systematic way you can list so you have all the sets?"

Equate elements of ordered pairs to find missing values, e.g. find $x, y$ where:

$$
(2 x, 1)=(3, y)
$$

Calculate the size of
Cartesian product of sets
Check by counting the elements of actual sets sets that the relation holds

Partition a given set
Given the partitions, of a set,

## elements in A

Describe the partition of a set as a grouping of its elements into non-empty subsets, in such a way that every element is included in exactly one subset

Guide students to partition sets in at least two different ways, e.g $S=\{1,2,3,4,5,6,7,8\}$
One possible partition is:
\{ 1 \}, \{2, 3, 4 \}, \{5, 6, 7, 8 \}
Another partition is:
$\{1,2\},\{3,4\},\{5,6,7$,
8 \}
etc.
Guide students to know and use the conditions
under which a set is partitioned, e.g. there should be no null set

Demonstrate and guide students to first find all the subsets of the given set, (we already know from SSS1, there are $2^{n}$ subsets)
e.g. $S=\{a, b, c\}$, we expect $2^{3}$ subsets, i.e. 8 subsets:
\{ \}, \{a \}, \{b \}, \{c \}, \{a,
b \},
write the original set
Write the power set of given sets

Calculate the size of the power set of given sets
$\{a, c\},\{b, c\},\{a, b, c\}$
Then guide students to write the power set $P$ as: $P=\{\{ \},\{a\},\{b\},\{c\}$, $\{a, b\},\{a, c\},\{b, c\},\{a$, $\mathrm{b}, \mathrm{c}\}$ \}

YEAR 2/TERM 3
PERMUTATIONS,
COMBINATIONS AND PROBABILITY

Fundamental Principles of Counting

Multiplication Principle: Factorial Notation

Permutations

Combinations
Probability of Events

Students will be able to:
Describe and apply the fundamental counting principles to solve simple problems

Calculate the permutation of $n$ objects using the multiplication principle

Calculate the permutations of a set of $n$ objects taken $r$ at a time

Define and calculate the number of combinations of a set of objects

Calculate the probability of an event occurring

Describe the multiplication principle of counting: if there are $m$ ways to do one task, and $n$ ways to do another task, then there are $m \times n$ ways to do both tasks (this can be extended to doing 3 or more tasks)

Ask pupils what this principle reminds them of (independent events in probability)

Demonstrate and guide students to state how many ways, using this principle, there are of, for example, throwing a six on a die and tossing a head on a coin, then verify by completing the sample space

Assist students to draw two-way tables and tree diagrams to help in enumerating all the outcomes from similar

## Textbooks

 Activity sheets Calculators ComputersInternet
Dice
Playing cards
Coins Spinners

Students are able to:
Identify what type of counting problem is in context

Use appropriately the multiplication and addition principles of counting to solve simple counting problem

Explain permutation to a peer
Use the multiplication principle to calculate the permutation of $n$ objects

Calculate the permutations of a set of $n$ objects taken $\underline{r}$ at a time

Solve real-life problems on permutation

Explain combination to a peer

Calculate the number of combinations of a set of objects

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experiments and from everyday life

Briefly introduce the addition principle of counting: if there are $m$ ways to do one task, and $n$ ways to do another
task, and we cannot do both at the same time, then there are $m+n$ ways to do both tasks (this can also be extended to doing 3 or more tasks)

Describe permutation as the arrangement of a number of objects, $n$, in order

Demonstrate using the multiplication principle to count the ways of ordering the letters $P$ and $Q-P Q$ and $Q P$, i.e., 2 or $2 \times 1$ ways

Guide students to do the same for three letters, $P$, $Q, R$ - there are 6 or $3 \times 2 \times 1$ ways

Assist students to
arrange 4 letters -
$4 \times 3 \times 2 \times 1$ or 24 ways
Therefore, to arrange $n$ letters (or objects) will

Solve real-life problems on permutation

Calculate the probability of an event
give $n!$ ways given by:
$n!=n(n-1)(n-2)(n-$
3)... $3 \times 2 \times 1$
read ' $n$ factorial', with
$0!=1$
Guide students to find the permutation of arranging $n$ different objects taken $r$ at a time, given by:
${ }^{n} P_{r}=\frac{n!}{(n-r)!}, r \leq n$
(Assume no replacement)
Demonstrate and guide students to use the appropriate button on their calculators to check their answers

Discuss how this formula compares with the multiplication principle

Describe combination as the selection of a number of objects in any order

Guide students to find the combination of selecting $r$
objects from $n$ given
objects, given by:
${ }^{n} C_{r}=\frac{n!}{r!(n-r)!}, r \leq n$
(Assume no replacement)
Review how to calculate the probability of an event from SSS1, i.e.

$$
P(E)=\frac{\text { number of outcomes of an event }}{\text { total number of outcomes }}
$$

Using examples, demonstrate and guide the students to calculate the required probability.

Assist pupils in using a combination of the multiplication principle, the permutation formula and the combination formula, depending on the context of the question, to find the total number of outcomes and the number of outcomes of an event
Assist students to understand a mapping pairs each element of a given set (the domain) with one or more elements of a second set (the range). They are usually represented by moping or arrow diagrams

Guide students to draw mapping diagrams to show the different types of mappings and how the elements are paired (e.g., one-to-one, onto, one-tomany, many-to-one, etc.)

## Textbooks

Activity sheets
Graphic calculators
Graph paper
Computers
Internet

Students are able to:
Take part in a group discussion on mappings, relations, and functions and their inter-relationships

Describe different types of mapping and their
representations
Draw mapping diagrams to represent relations and functions

Explain what functions are and use the correct notations to describe them
algebraically
as sets of ordered pairs
Find the domain and range of a function

Find inverse functions
Find composite functions

Guide students to understand a relation as a collection of ordered pairs, ( $x, y$ ), which are related by a rule and represented by a mapping diagram

Functions are relations which pairs one element in the domain with only one element in the range

Describe the elements in the domain of a function as the set of independent variables, the input values, which the function processes; and the range as the set of dependent variables, the output values, which the function generates

Guide students to use the notation $x$ for the elements in the domain, and $f(x)$ (read as 'function of $x^{\prime}$ or ' $f$ of $x^{\prime}$ ', as the function which generates the elements in the
range. E.g.
$f(x)=x^{2}$
read:
' $f$ of x equals x squared'
Other letters are used which denote the same thing, e.g., $g(x)$ ('g of $x$ ')

Research similarities and differences between
mathematical and
programming functions
Make a presentation to their peers on the use of mathematical functions in computer programming

Identify given functions as linear, quadratic,
trigonometric, etc., or not a function

Represent given functions in a variety of formats

Find the domain and range of a given function

Find the inverse of a given function

Find the domain and range of the inverse of a given
function
find: fog, gof,
$(f \circ g)^{-1}$, $f^{1} \mathrm{og}^{-1}$, etc.
Find the domain and range of the composite of a given function

## or $h(x)$

Discuss the similarities and differences between mathematical functions and programming
functions, (e.g., both
types of function accepts input, does some processing and generates
an output)
Discuss why and how mathematical functions are used in computer
programming
Discuss types of functions, e.g.,

- linear functions
- quadratic functions
- higher-order polynomial functions
- rational functions
- logarithmic functions
- exponential functions
- trigonometric functions
- etc.

Discuss how to identify a relation which is not a function, e.g., equation (graph) of a circle

Describe each type of function and assist students to give the general formula and
examples of each type Demonstrate and guide students to represent functions in each of the ways given, including using graphic calculators and computers

Provide a selection of functions in one format and guide students to represent them in one or more of the other formats, including as a set of ordered pairs

Demonstrate and guide students to find the domain and range of functions given in a variety of formats

Demonstrate and guide students to find the
inverse, $f^{1}(x)$ of a
function, $f(x)$ given in a
variety of formats, e.g.
if $f(x)=2 x$, then
$f^{1}(x)=\frac{1}{2} x, x \in \mathbb{R}$
Guide students to make $x$ the subject of more complex functions using $y=f(x)$ for convenience
for $f(x)=3 x+2$
rewrite $y=3 x+2$
and make $x$ the subject of the equation

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Discuss a composite
function as a combination
of two or more functions
Guide students to find the composite of two
functions $f(\mathrm{x})$ and $g(\mathrm{x})$ as:
$f \circ g(x)=f(g(x)]$
which is $g(x)$ followed by
$f(x)$
and $g \circ f(x)=g[f(x)]$
which is $f(x)$ followed by
$g(x)$
Assist students to find the domain and range of the composite function

Guide students to find the
inverse of composite
functions, as well as the
composite of inverse
functions

YEAR 3/TERM 1

## MATRICES

Basic Matrices Concepts
Addition and Subtraction of Matrices

Scalar Multiplication
Matrix Multiplication
Properties of Matrix Operations

Determinant of Matrices
Matrix Row Operations Inverse Matrices

Students will be able to:
Identify a matrix, stating its order and type

Explain the use of matrices in computer programming

Identify equal matrices and find missing elements

Find the transpose of a matrix

Find the sum and difference of two matrices

Find scalar multiples of a matrix

Find the product of two matrices

Understand and use the properties of matrix operations:

- commutative
- associative
- distributive
- additive and multiplicative identities
- additive and multiplicative inverses

Find the determinant of a matrix

Use examples of matrices to guide students to identify a matrix as a rectangular arrangement (called array) of items, usually numbers, into rows and columns. Show the notation for matrices and how elements of a matrix are identified

Demonstrate and guide students to show how matrices are written and how to determine the order or dimension ( $m \mathrm{x}$ $n$ ) and the notation for an element, $\mathrm{a}_{\mathrm{ij}}$

Give examples of each type a matrix, e.g., square, triangular, row column, identity, zero, etc.

Ask students to write matrices for their peers to give the order and type. Students can also, give order and type for their peers to give examples

Discuss why matrices are important in computer programming

- Textbooks
- Activity sheets
- Computers
- Internet

Students are able to:
Identify a matrix with $m$ rows and $n$ columns as of order $m$ $\mathrm{x} n(m$ by $n$ )

Write a matrix using the correct notation

Identify elements of a matrix using the correct notation, $a_{i j}$

State the type of matrix depending on the number of rows, columns present

Challenge their peers to identify the order and type of a variety of matrices

Answer challenges from their peers to identify the order and type of a variety of matrices

Explain why matrices are important to computer programming

Identify equal matrices when given a variety of matrices

Use equal matrices to find missing elements in a matrix

Find the transpose of given

Guide students to use the internet to research and discuss matrices and how it is mainly used in computer programming to solve systems of linear equations, e.g., in graphics and image processing

Use examples of matrices to demonstrate and guide students to identify equal matrices as having the same order, with the corresponding elements of the two matrices equal

Use that fact to guide students to find missing elements in either matrix

Guide students how to find the transpose of an $m \times n$ matrix A by switching the rows and columns of the matrix,
i.e., to get an $n \times m$ matrix,
$\left(A^{\top}\right)_{\mathrm{ij}}=\mathrm{A}_{\mathrm{j}}$
Demonstrate using examples how two matrices, A, B are added and subtracted provided they are of the same order:
$(A+B)_{i j}=A_{i j}+B_{i j}$
and $\quad(A-B)_{i j}=A_{i j}-B_{i j}$

## matrices

Check the order of matrices, then add or subtract as required

Solve equations involving addition and subtraction of matrices

Perform multiplication of a given matrix by a given scalar

Give the condition for matrix multiplication to occur

Find the product of two matrices

Verify one or more of the properties using examples of matrices

Investigate which matrix operations do not follow the properties and why

Explain how to find the determinant of a $2 \times 2$ matrix

Use their algorithm to find the determinant of a $2 \times 2$ matrix

Extend the algorithm for finding determinants of $2 \times 2$ matrices to find the determinant of $3 \times 3$ matrices

Find the inverse of a given $2 \times 2$ matrix using the standard algorithm

Find the inverse of a given $3 \times 3$ matrix using the extended algorithm

Perform and use the correct notation for single row
operations on a matrix, e.g.
$R_{2} \square R_{1},-3 R_{3} \square R_{3}, R_{1}+R_{3} \square$
$R_{3}$
Perform and use the correct notation for multiple row operations on a matrix, e.g. $2 R_{1}-4 R_{3} \square R_{3}$
$A+B=B+A$
associative
$(A+B)+C=A+(B+C)$

- additive identity
$A+0=0+A=A$
for a unique $m \times n$ matrix,
0
- additive inverse
$A+(-A)=0=(-A)+A$ where $-A$ is a unique $m x$ $n$ matrix

Guide students to use matrices to prove these properties

Demonstrate using a $2 \times 2$ matrix, $A$, the algorithm to find the determinant, symbolised by $\operatorname{det}(A)$ or
$|A|$, of the matrix
Guide the students to write their own algorithm using pseudocode and flowchart of how to find the determinant of a matrix.

Guide them to exchange their algorithms with each other use to find determinants of $2 \times 2$
matrices
Guide students to extend their algorithm to finding the determinant of $3 \times 3$
matrices

Share and discuss a few of the algorithms and make improvements

Define the inverse, $A^{-1}$, of the matrix, A , as:
$A \times A^{-1}=A^{-1} \times A=1$
where $I$ is the identity
matrix
Demonstrate and guide students to find the inverse of a matrix using the determinant and the standard algorithm

Guide students to extend the algorithm as demonstrated to find the determinant of $3 \times 3$ matrices

Demonstrate and guide students to perform row operations on a matrix using the three
operations:

- switching rows
- multiplying a row by a non-zero number
- adding rows

Demonstrate the notation for showing the row operation. For example:

- switching rows 1 and 3
is written as $R_{1} \square R_{3}$
- multiplying row 2 by 3 is written as $3 R_{2} \square R_{2}$

Guide students to perform these operations iteratively and in combination till they get the required matrix
Discuss why matrices are used to solve systems of linear equation

Use a system of linear equations in two variables and demonstrate how it can be represented as a matrix by using the coefficients of each equation to form a row of the matrix

Guide students to make sure that both equations are in the linear equation form:
$a x+b y=c$
before they are written in matrix form:
$A x=B$
where $A, B, x$ are matrices

Review how to find determinants and inverse of a matrix.

Textbooks Activity sheets
Explain why matrices are used to solve systems of linear equations

Represent a linear system as a matrix

Solve a $2 \times 2$ system of linear equations using an inverse matrix

Solve a $3 \times 3$ system of linear equations using an inverse matrix

Recognise the Gaussian elimination as an algorithm used to find the solution of a system of linear equations in $n$ variables

Write the augmented matrix of a system of equations

Write the system of equations from an augmented matrix

| Perform row operations on a matrix | Using the matrix representation: |
| :---: | :---: |
| Solve a $2 \times 2$ system of linear equations using Gaussian elimination | $A x=B$ <br> guide students to find the unknown variables as: |
| Solve a $3 \times 3$ system of linear equations using | $x=A^{-1} b$ |
| Gaussian elimination <br> Write an algorithm to solve a system of linear equations in $n$ variables using Gaussian elimination | Demonstrate and guide students to perform a matrix multiplication of $A^{-1}$ and $b$, and equate the resulting matrix with the unknown variables |
|  | Guide students to solve a system of $3 \times 3$ linear equations using an inverse matrix |
|  | Discuss how the Gaussian elimination method uses matrices to solve systems. Show how the steps follow an algorithm (which can be written as a program) |
|  | Demonstrate and guide students to write the system as an augmented matrix |

Demonstrate and guide students to also be able to write a system from an augmented matrix

Review how to perform row operations till they get the required matrix (an upper triangular matrix with all the element in the main diagonal equal to 1 )

Demonstrate and guide students to solve a $2 \times 2$ system of linear equations by following the algorithm:

- represent the linear system as a matrix
- write the augmented matrix of the system of equations
- perform row operations on the matrix to get the required matrix
- use back substitution to find the solution for each variable in the system


## Guide students to use

Gaussian elimination to

# solve a $3 \times 3$ linear system <br> Guide students to write <br> pseudocode and <br> flowchart to solve a $2 \times 2$ <br> system of linear <br> equations using Gaussian <br> elimination <br> Extend the algorithm to <br> an $n$ by $n$ system 

## RESOURCES

Textbook
Information sheets on the use of non-decimal number systems in computers
Activity sheets
Computers / Smart Phones / Calculators
Internet
Objects or pictures of objects
2D attribute shapes
Sets of number cards with common attribute e.g., odd, even, prime, square
Sets of cards with names e.g., places, capital cities, flowers, surnames starting with same initial

