# The New Senior Secondary Curriculum for Sierra Leone 

## Subject syllabus for Further Mathematics <br> Subject stream: Mathematics and Numeracy



This subject syllabus is based on the National Curriculum Framework for Senior Secondary Education. It was prepared by national curriculum specialists and subject experts.

## Curriculum elements for Further Mathematics - a core subject

## Subject Description

The mathematics program will equip students with the basic competencies that would help their prospects of employment and enable them to use mathematics to solve real-life problems. Mathematics is invaluable due to its widespread application in every sphere of life, in the fields of science, technology, agriculture, economics and other social activities. This subject seeks to train students with competencies that will build on the teaching and learning of mathematics at Junior Secondary Level. The program focuses on having every student as its target.
It aims at educating all students to be mathematically literate and more capable in utilizing the available resources and opportunities to improve quality at work, their welfare, and prosperity.

## General Learning Outcomes (Broad Goals)

To develop in all students the capabilities and skills to:

- Use Mathematics as a language to analyse and communicate information and ideas
- Use computational and analytic skills for practical use
- Identify mathematical concepts in their fields of study
- Identify mathematics as a tool and as a discipline
- Cooperative/work together with other students to carry out activities and projects in mathematics and consequently acquire the values of cooperation, tolerance and diligence


## Subject Content Outline by Broad Themes \& Specific Topics

## Sets

- Describe set and the various types.
- Apply the algebra of sets.
- Solve two and three set problems (including use of Venn diagrams)

Surds

- Four operations on surds $(+,-, \times \& \div)$
- Rationalise the denominator (including binomial denominators)


## Relations and Functions

- Relations, Mappings and Functions
- Function Notation.
- Types of functions
- Representing functions.
- Inverse Functions and Composite Functions
- Graphs and roots of Functions


## Polynomial Functions

- General Characteristics
- Linear function
- Quadratic Function
- Cubic Functions
- Rational Functions
- Partial fraction
- Exponential function
- Logarithmic Function


## The Binomial Theorem

- Binomials expansion by Binomial theorem.
- Binomial expansion by Pascal's triangle


## Sequences and Series

Finite and Infinite sequences

- Recurrence sequence
- Arithmetic sequence and Geometric sequence

Finite and infinite series

- Arithmetic series and Geometric series
- Sum of Arithmetic sequences and Geometric sequences


## Co-ordinate Geometry

- Loci
- Straight Lines
- Circles
- Parabolas


## Trigonometry

- Trigonometric Ratios and Rules
- Compound Angles
- Multiple Angles
- Trigonometric Functions


## Calculus

## Limits

Introduction to Derivatives
Differentiation

- Methods of Differentiation
- Implicit Differentiation
- Derivative of Trig Functions

Applications of Differentiation
Integration

- The general solution of (Indefinite integral and Definite integral)

Techniques of integration

- Integration of Trigonometric Functions.
- Integration by substitution
- Integration of Logarithmic functions
- Integration of exponential functions.

Applications of Integration

- Area under curves
- Numerical integration (Trapezoidal Rule)


## Vectors

Vectors and scalars

- Properties of vectors (representing vectors, equal vectors, null or zero vector)
- The magnitude and direction of a vector
- Algebra of vectors
- Triangle law of vector addition


## Statics

Resultant and resolving forces into components

- Equilibrium of coplanar forces
- Types of forces (weight, tension and trust)
- Friction and coefficient of friction


## Kinematics of a particle

Dynamics

- Rotational motion of rigid bodies (moment of inertia of a particle and rigid body)
- Newton's laws of motion
- Motion of two connected particles (problems involving pulleys)
- Momentum and impulse (principle of conservation of momentum)
- Sum of moments
- Equilibrium of a lamina under parallel forces (non-uniform rods)


## Matrices

- Operations on matrices
- Finding the determinant and inverse of a matrix (limited to $2 \times 2$ matrices)
- Application of matrices (Cramer's rule) to solve simultaneous linear equations in two variables


## Linear Transformations

- The concept of linear transformation
- Images of points under given linear transformation
- Determine the matrices of linear transformation
- The inverse of linear transformation
- Composition of linear transformation


## Statistics

- Data Representation
- Data Analysis
- Correlation


## Probability

- Introductory concepts
- Permutation and Combination
- Binomial Probability Distribution.

|  | SSS 1 | SSS 2 | SSS 3 |
| :---: | :---: | :---: | :---: |
| Term 1 | Sets <br> Describe set and the various types. Apply the algebra of sets. Solve two and three set problems (including use of Venn diagrams <br> Surds <br> Four operations on surds (,,$+- \times \& \div$ ) Rationalise the denominator (including binomial denominators) | The Binomial Theorem <br> Binomials expansion by Binomial theorem. <br> Binomial expansion by Pascal's triangle <br> Sequences and Series <br> Finite and Infinite sequences <br> - Recurrence sequence <br> - Arithmetic sequence and Geometric sequence <br> Finite and infinite series <br> - Arithmetic series and Geometric series <br> - Sum of Arithmetic sequences and Geometric sequences | Vectors <br> Vectors and scalars <br> Properties of vectors (representing vectors, equal vectors, null or zero vector) <br> The magnitude and direction of a vector <br> Algebra of vectors <br> Triangle law of vector addition <br> Statics <br> Resultant and resolving forces into <br> components <br> Equilibrium of coplanar forces <br> Types of forces (weight, tension and trust) <br> Friction and coefficient of friction <br> Kinematics of a particle <br> Dynamics <br> moment of inertia of a particle and rigid body <br> Newton's laws of motion <br> Motion of two connected particles <br> Momentum and impulse <br> Sum of moments <br> Equilibrium of a lamina under parallel forces |
| Term 2 | Relations and Functions <br> Relations, Mappings and Functions Function Notation. <br> Types of functions <br> Representing functions. <br> Inverse Functions and Composite <br> Functions <br> Graphs and roots of Functions | Co-ordinate Geometry <br> - Loci <br> - Straight Lines <br> - Circles <br> - Parabolas Trigonometry <br> - Trigonometric Ratios and Rules <br> - Compound Angles | Matrices <br> Operations on matrices Finding the determinant and inverse of a matrix (limited to $2 \times 2$ matrices) <br> Application of matrices (Cramer's rule) to solve simultaneous linear equations in two variables |



Term 3

## Polynomial Functions

- General Characteristics
- Linear function
- Quadratic Function
- Cubic Functions
- Rational Functions
- Partial fraction
- Exponential function
- Logarithmic Function
- Multiple angles
- Trigonometric functions


## Calculus

- Differentiation
- Applications of Differentiation
- Integration
- Applications of Integration


## Linear Transformations

The concept of linear transformation Images of points under given linear transformation
Determine the matrices of linear transformation
The inverse of linear transformation
Composition of linear transformation

## Statistics

- Data Representation
- Data Analysis
- Correlation

Probability

- Introductory concepts
- Permutation and Combination
- Binomial Probability Distribution


## Teaching Syllabus

Development of Curriculum \& Syllabuses for Senior Secondary Education in Sierra Leone

| Topic/Theme/Unit | Expected learning outcomes | Recommended teaching methods | Suggested resources | Assessment of learning outcomes |
| :---: | :---: | :---: | :---: | :---: |
| Sets <br> Understanding and applying the algebra of sets. <br> Solving two- and threeset problems | Students should be able to: <br> - Describe set and the various types. <br> - Apply the algebra of sets. <br> - Solve two and three set problems (including use of Venn diagrams) | YEAR 1/TERM 1 <br> Introduce set as very important concept in mathematics in everyday life collection. <br> Discuss with the students the definition of set as a well-defined collection of objects of the same kind. <br> Explain the various types of sets <br> Universal set, empty set, subset. <br> Teacher to Describe set notation, members/elements by <br> - Listing its members $\mathrm{T}=(2,3,4,5,7,11)$ <br> - Given word description of its members <br> $A=$ (prime numbers less than 12). <br> - Using a set-builder notation $\mathrm{B}=(x: 1<x<12)$ <br> where $x$ is a prime numbers Explain the various types of sets | Diagram of various set type on vanguard <br> Illustrated Venn diagram on vanguard | Ask student short answer questions <br> Eg. Name any 3 types of set. <br> Write two sets and ask students to illustrate union, intersect and complement of set. <br> Write a three sets word problem on the board and asked the students to calculate <br> i). One only <br> ii) Both. <br> iii) All the three. |


|  |  | Teacher to explain the illustrated Venn diagram on the vanguard <br> Discuss operation of sets <br> U.....Union <br> ก..... Intersection <br> !...... Complement <br> Disjoint sets <br> Solve two and three set problems (including use of Venn diagrams) |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Surds Performing the four operations on surds. Rationalising the denominator | Students should be able to: <br> - Perform the four operations on surds (,,$+- \times \& \div$ ) <br> - Rationalise the denominator (including binomial denominators) | Review the concept of perfect squares <br> Discuss with the students how a multiple number is simplify into two factors <br> Eg. $\sqrt{500}=\sqrt{100 \times 5}$ $\begin{aligned} & =\sqrt{100} \times \sqrt{5} \\ & =10 \sqrt{5} \end{aligned}$ <br> Solve problems with the students involving addition, subtraction, multiplication and division of surds <br> Demonstrate to the students how to Rationalise denominators | Table of perfect square roots. $\begin{aligned} & \sqrt{4}=2 \\ & \sqrt{9}=3 \\ & \sqrt{16}=4 \\ & \sqrt{25}=5 \end{aligned}$ <br> etc | Class exercises <br> Eg. <br> Simplify $\sqrt{225}, \sqrt{243}$ Etc. <br> Evaluate and leave your answer in $a \sqrt{b}$ <br> a). $\sqrt{50}+\sqrt{18}$ <br> b). $\sqrt{847}-\sqrt{175}$ <br> Rationalize the denominator |

## YEAR 1/TERM 2

## Relations and <br> Functions

Describing Relations,
Mappings and Functions
Using Function Notation

Specifying the different types of functions

Representing functions.

Inverse Functions and Composite Functions

Graphs and roots of
Functions

## Students should be able

 to:- Describe Relations, Mappings and Functions
- Apply Function Notation, (domain, range, dependent and independent variables)
- Identify the different types of functions (one to one, one many \& many to one etc.)
- Represent functions using tables, Algebraically, Graphically
- Evaluate Inverse and Composite Functions
- Draw Graphs of Functions and determine the Roots of Functions

Introduce the concept of relation as an association that exits between two sets of objects the domain and codomain.

Further explain that relation can be shown by means of order pairs

Discuss with the students that the set of all possible images of the domain is called range.

Note! A relation may exist between two sets but not all the elements of the domain may be associated with elements of the co-domain. Types of relation One -to-one, many- to- one, many -to- many

Describe Mapping as a relation in which each member in the domain maps onto only one member in the co-domain
l.e one to one and many to one relation are mappings.

Discuss with the students how to identify functions from these characteristics.

Illustrated cases of relation, mapping and functions on vanguard

- Graph board
- Graph paper
- Blackboard ruler
- Foot rule
- Markers
- Colored chalks
- Pencils

Diagram of the various types of functions

- One to one
- One to many
- Many to one

Showing the Domain and the image (Range)

Ask students to draw the various types of functions


Eg. Find the images of the elements of the domain [-2,-
$10,1,2$ ] define by the
function $f: \mathrm{x} \rightarrow \frac{3 x-1}{x-3}$
Eg. Draw the graph of the function $\mathrm{f}(\mathrm{x})=2 \mathrm{x}+1$ in the interval $-2 \leq x \leq 4$

## Inverse Function

Given the function
i). $f(x)=3 x-2$, find its inverse.
ii). Given $f(x)=2 x+3$,
find $f^{-1}(x)$.
iii). Find the inverse of the following function
$g(x)=(x+4) /(2 x-5)$

## Composite Function

i). Given the functions
$f(x)=x^{2}+6$ and $g(x)=$
$2 x-1$, find ( $f \circ g$ ) ( $x$ ).
ii). Given the functions
a). Each element in A must be matched with an element in B b). Some elements in B may not be matched with any element in A
c). Two or more elements in A may be matched with the same element in B
d). An element in A cannot be matched with two different elements in B.

Discuss with the students the domain and range of the given function.

Illustrate Graphs of functions using the graph board. If $f$ is a function with domain $D$, then the graph of ' $f$ ' is the set of all points $P(x, f(x))$ in the plane. That is the graph of ' $f$ ' is the graph of $y=f(x)$.

Solve problems with the students involving function of a function
ie $(f \circ g)(x)=f(g(x))$ and
inverse function
Restrict to simple algebraic
functions only.
Draw graphs of Functions and determine the Roots of Functions using the graph board

YEAR 1/TERM 3

Polynomial Functions 1

General Characteristics of functions

## Polynomial Functions

 11Linear Functions

## Students should be able

 to:- Recognise equations of polynomial functions of degree $\leq 4$
- Simplify the algebra of polynomial functions
- State and apply the Remainder theorem and the Factor theorem


## Students should be able

 to:- identify linear function represented by a straight-line graph
- Sketch graphs of linear equations
- Derive equations of linear equations using slopeintercept, slope point, two points
- Find equations of parallel and
write the remainder and factor theorem and demonstrate how to apply them in
simplifying polynomia
Remainder Theorem
if a polynomial $f(x)$ is divided
by $x-k$, the remainder is

$$
r=f(k)
$$

Eg. Use the remainder theorem to evaluate the
function at $\mathrm{x}=-2$
$f(x)=3 x^{3}+8 x^{2}+5 x-7$

## Factor Theorem

A polynomial $f(x)$ has a
factor $(x-k)$ if and only if
$f(k)=0$
Eg. Show that $(x-2)$ and $(x+$ 3)
are factors of
$f(x)=2 x^{4}+7 x^{3}-4 x^{2}-27 x-18$
Discuss linear function as a graph
$f(x)=a x+b$ is a line with slope $m=a$ and $y$ - intercept at $(0, b)$.

Teacher Use the graph board,
Blackboard ruler, Colored chalks and allow students to work on graph paper to demonstrate how to sketch linear graph

Help the students Derive equations of linear equations

## Textbooks

Chart showing polynomial
functions of degree $\leq 4$
a). Linear function
b). Quadratic function
c). Cubic function
). The remainder after

Graph board
Graph paper Blackboard ruler
Foot rule
Markers
Colored chalks
Pencils
$2 x^{2}-5 x-1$ is divided by $x-3$
ii). the remainder after $2 x^{2}-5 x-1$ is divided by $x-5$
$2 x^{2}-5 x-1$ is divided by $x-5$
iii). Use the Factor Theorem to find the zeros of $\boldsymbol{f}(\boldsymbol{x})=$ $x^{3}+4 x^{2}-4 x-16$ given that $(x-2)$ is a factor of a polynomial.
iv. use the factor theorem to
find the zeros of $f(x)=x^{3}-$
$6 x^{2}-x+30$. Given that $(x+2)$
find the zeros of $\boldsymbol{f}(\boldsymbol{x})=\boldsymbol{x}^{3}-$
$6 x^{2}-\boldsymbol{x}+\mathbf{3 0}$. Given that $(\mathrm{x}+2)$
is a factor of a polynomial.

Plot the points and find the slope of the lie that passes through the pair of points
i). $(-3,-)$ and $(1,6)$
ii). $(2,4)$ and $(4,-4)$

Use the point on the line and the slope of the line to determine the general equation of the line.

1. Point ( 2,1 ) and slope $\mathrm{m}=1$
2. Point $(-5,4)$ and slope $m=2$

Graph board
perpendicular lines to a given line

- Solve simultaneous linear equations graphically or algebraically
- sketch and solve linear inequalities
- Apply linear inequalities to Linear Programming
using
a) slope-intercept

$$
m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$

b) point - slope
$y-y_{1}=m\left(x-x_{1}\right)$
c) two points
$D(x, y)$ and $R(x, y)$
Parallel line $\leftrightarrow m_{1}=m_{2}$

## Perpendicular line

$m_{1} m_{2}=-1$
Where $m_{1} m_{2}$ are gradients of the two lines?

Teacher Use the graph board, Blackboard ruler, Colored chalks and allow students to work on graph paper to demonstrate how to sketch simultaneous linear equations graphically and algebraically (including methods of elimination and substitution

Explain symbols involve in
Sketching and solving linear
inequalities ( $<,>, \leq, \& \geq$ )
Linear Inequalities
e.g. $2 x+5 y \leq 1, \quad x+3 y \geq 3$

Apply linear inequalities to Linear Programming (optimisation, objective function, constraints and feasible solution)

Solve practical problems to maximize profit.

Graph paper Blackboard ruler

## Foot rule

Markers
Colored chalks
Pencils

Determine whether the lines $\mathrm{L}_{1}$ and $\mathrm{L}_{2}$ are parallel or perpendicular
i). $L_{1}(0,-1),(5,9)$
$L_{2}(0,3),(4,1)$
. $L_{1}(3,6),(-6,0)$
$L_{2}(0,-1),\left(5, \frac{7}{3}\right)$

## Simultaneous Linear

Solve the following pair of simultaneous linear
equations: $2 x+3 y=8$ $3 x+2 y=7$ Using elimination, substitution and graphical methods.

## Inequalities

Solve the inequalities:

1. $2(x-4) \geq 3 x-5$
2. $7 x+11>2 x+5$
3. $2(x+3)<x+1$
4. $-5 \leq 2 x-7 \leq 1$

Quadratic Functions Students should be able to:

- Recognise quadratic functions represented by a parabola
- Sketch graphs of quadratic functions using turning points, intercepts and axis of symmetry
- Determine the nature of the roots of a quadratic equation Use the discriminant
- Solve quadratic equations by
- Graphical method
- Factorizing method
- Completing the square
- Quadratic formula
- Derive quadratic equations given sufficient information
- Solve simultaneous equations for one linear, one quadratic
- Extend concepts to sketching and solving quadratic inequalities

Teacher to define quadratic function. Let $a, b$, and $c$ be real numbers with $a \neq 0$. The function $f(x)=a x^{2}+b x+c$

Use the graph board, blackboard ruler to illustrate quadratic graph turning points, intercepts and axis of symmetry.

Solve problem on quadratic equation by
a) Graphical method
b) Factorizing method
c) Completing the square
d) Quadratic formula
$x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
Demonstrate the Roots of quadratic equations - equal roots ( $b^{2}-4 a c=0$ ), real and unequal roots ( $b^{2}-4 a c>0$ ), imaginary roots ( $b^{2}-4 a c<0$ ); sum and product of roots of a quadratic equation
e.g. if the roots of the equation $3 x^{2}+5 x+2=0$ are $\alpha$ and $\beta$, form the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Solving quadratic inequalities

Solve the quadratic equation by completing the square
I. $x^{2}+4 x+1=0$

Solve the quadratic equation by formula method
$-4 x^{2}+x+3=0$.
Solve the quadratic equation by graphical method.
i. $-4 x^{2}+x+3=0$.

## Cubic Functions

Polynomial Functions 111
Rational Functions

Students should be able to:

- Recognise cubic functions as functions of degree 3
- Draw graphs of cubic functions for a given range
- Factorise and solve cubic equations


## Students should be able

 to:- Recognize rational function as a quotient of two polynomial functions
- Apply the four operations on rational functions

Discuss cubic functions as functions of degree 3
e.g. f: $x \rightarrow a x^{3}+b x^{2}+c x+d$. teacher should support students to Draw graphs of cubic
functions for a given range. Explain how to Factorize cubic expressions and solution of cubic equations.
Factorization of $a^{3} \pm b^{3}$

Teacher to explain to the students that rational function can be written in the form

$$
f(x)=\frac{N(x)}{D(x)}
$$

Where $N(x)$ and $D(x)$ are polynomials and $D(x)$ is not zero.

Solve problems as work examples with the students involving rational functions Eg. Find the domain of the function

$$
f(x)=\frac{4(x+1)}{x(x-4)}
$$

Decompose into partial fraction

$$
f(x)=\frac{N(x)}{D(x)}
$$

Eg. Write the partial fraction decomposition of

## Cubic Function

Determine the roots of the
cubic equation
$2 x^{3}+3 x^{2}-11 x-6=0$
Find the roots of the cubic equation
$x^{3}-6 x^{2}+11 x-6=0$
Solve the cubic equation
$x^{3}-23 x^{2}+142 x-120$
Find the roots of $x^{3}+5 x^{2}+$ $2 x-8=0$ graphically.

Charts of laws of indices Chart of laws of logarithms Graph board
Graph paper
Blackboard ruler
Foot rule
Markers
Colored chalks
Pencils
1). If $f: x \rightarrow \frac{1}{2+x}$, find the range if the domain is the se $[x: 1 \leq x \leq 5]$
2). Simplify the following rational functions
$\frac{1}{x-2}+\frac{3}{x+1}$
$\frac{4}{x+2}-\frac{3}{x+3}$
$\frac{2 x}{x^{2}-1} \div \frac{x^{2}-2 x}{x^{2}-2 x+1}$

Resolve $\frac{11-3 x}{x^{2}+2 x-3}$ into partial fractions.

Resolve $\frac{x^{2}-1}{x^{2}-3 x+2}$ into partial fractions.
b) Repeated linear factors in the denominator
c) Quadratic factors in the denominator

Exponential and Logarithmic Functions

## Students should be able

to:

- Apply the laws of indices
- Solve equations involving indices
- Apply the laws of logarithms
- Solve equations involving logarithm and change of base
- Draw and interpret graphs of exponential relations

$$
f(x)=\frac{x+7}{x^{2}-x-6}
$$

Discuss with the students relation between exponential and indices.
i.e Exponential function $\boldsymbol{f}$ with base $\boldsymbol{a}$ is denoted by

$$
f(x)=a^{x}
$$

Where $\mathrm{a}>0, \mathrm{a} \neq 1$ and $x$ is any real number.
*Note to the students that in many application the most convenient choice for a base is the irrational number $e=$ 2.718281828

Discuss the definition of logarithms function with base a.
le for $x>0$ and $0<a \neq 1$ $y=\log _{a} x$ if and only if $x=$ $a^{y}$
Hence $f(x)=\log _{a} x$ is the logarithms function with base a.

Eg. Simplify $\log _{5} 5^{x}$
Solve problems with students involving exponential (indices) and logarithm equations
Eg. Solve $2\left(3^{2 x-5}\right)-4=11$
Solve $\log _{3}(5 x-1)=\log _{3}(x+7)$

Without using mathematical table simplify the following
1). $\left(\frac{16}{81}\right)^{-\frac{3}{4}}$
ii). $16^{-\frac{3}{2}}$

Find the value of $x$ in the
following
i). $3^{x^{2-1}}=9^{4}$
ii) $3^{2 x}-4\left(3^{x}\right)+3=0$

Simplify the following
i). $\log _{5} 10+\log _{5} 12$
ii) $\log _{3} 24+\log _{3} 15-\log _{3} 10$

Solve the following equation
$\log _{10}(5 x+6)=\log _{10}(5 x-6)$
ii). $\log _{10}\left(x^{2} 1\right)-2 \log _{10} x=1$

Demonstrate the properties of
logarithms.

$$
\begin{aligned}
& \log _{a}(U V)=\log _{a} U+\log _{a} V \\
& \log _{a}\left(\frac{U}{V}\right)=\log _{a} U-\log _{a} V \\
& \log _{a} U^{n}=n \log _{a} U
\end{aligned}
$$

*Note to the students that there is a natural logarithmic function defined by

$$
f(x)=\log _{a} x=\ln x \quad x>0
$$

## YEAR 2/TERM 1

## The Binomial Theorem

Use of the binomial theorem for positive integral index only.

Proof of the theorem not required

## Students should be able

 to:- Expand powers of binomials using the binomial theorem.
- Generate coefficient of binomial expansion by Pascal's triangle.

Discuss the binomial theorem with the students which state that for
$(x+y)^{0}=1$
$(x+y)^{1}=x+y$
$(x+y)^{2}=x^{2}+2 x y+y^{2}$
For any $(x+y)^{n}$
$(x+y)^{n}=x^{n}+n x^{n-1} y+$
$\cdots+C_{r}^{n} x^{n-r} y^{r}+\cdots$
Illustrate the Pascal's triangle to generate coefficient of binomial expansion
$(x+y)^{n}$ where $\mathrm{n}=$
0,1,2,3,4 $\ldots$
Demonstrate with the students work examples on binomial expansion using both methods.

Use the binomial series to determine the expansion of $(2 a-3 b)^{5}$

Use the binomial series to determine the expansion of $(2+x)^{7}$

Use Pascal's triangle to expand $(2-y)^{7}$

Expand $(2 a-3 b)^{5}$ using Pascal's triangle

Determine, using Pascal's triangle method, the expansion of $(2 p-3 q)^{5}$

## Eg.

a). Write the binomial expansion for the
expression $(x+1)^{3}$
b). Find the binomial
coefficient

$$
(x+1)^{4}
$$

## Sequences and Series <br> Students should be able

 to:- Differentiate between finite and infinite sequences
- Describe the properties of Recurrence sequences, Arithmetic sequences, Geometric sequences
- Differentiating between finite and infinite series
- Describe the properties of Recurrence series, Arithmetic series, Geometric series
- Calculate sum of Arithmetic sequences and Geometric sequences

Teacher to explain finite and infinite sequences.

Illustrate how to find terms of a sequence
$a_{1}, a_{2}, a_{3}, a_{4}, \ldots a_{n}$ Discuss the properties of sequences.

## a). Recurrence sequences

Generating the terms of a recurrence series and finding
an explicit formula for the sequence e.g. $0.9999=\frac{9}{10}+$ $\frac{9}{10^{2}}+\frac{9}{10^{3}}+\frac{9}{10^{4}}+\cdots$

## b). Arithmetic sequences

A sequence whose
consecutive term have a
Common Difference

$$
a_{n}=d n+c
$$

$\mathrm{Un}=\mathrm{U} 1+(\mathrm{n}-1) \mathrm{d}$
c). Geometric sequences

A sequence whose
consecutive terms have a
Common Ratio.
$a_{n}=a_{1} r^{n-1}$

## Class excises

AP
example, the sum of the first seven terms of the series 1 , $4,7,10,13$,.

Determine the number of the term
whose value is 22 in the
series $2 \frac{1}{2}, 4,5 \frac{1}{2}, 7, \ldots$.
Find the sum of the first 12 terms of the series $5,9,13$ 17, ...

GP
example, find the sum of the first eight terms of the GP
$1,2,4,8,16, \ldots$
Find the sum to infinity of the series $3,1,1 \frac{1}{3}, \ldots$

Find the tenth term of the series $5,10,20,40$,.

Solve problems involving finite and infinite sequence.

Demonstrate the step by step method of calculating sum of Arithmetic sequences and Geometric sequences

- Sum of Arithmetic series (AP)
$S_{n}=\frac{n}{2}\left(a_{1}+a_{n}\right)$
$S_{n}=\frac{n}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]$
- Sum of Geometric series (GP)
$\boldsymbol{S}_{\boldsymbol{n}}$
$=\sum_{n=1}^{n} a_{1} r^{n-1}=a_{1}\left(\frac{1-r^{n}}{1-r}\right)$
when $r<1$
$s_{n}=\frac{a_{1}\left(r^{n}-1\right)}{(r-1)}$ when $\mathrm{r}>1$


## YEAR 2/TERM 2

## Co-ordinate Geometry

Loci
Equation to a locus.

Students should be able to:

- Describe locus of a point
- Sketch the locus of points satisfying given conditions
State the Locus theorem and how it can be used in real life situations or activities. Determine the locus

Sketch the locus of points satisfying given conditions i). The equation of a curve is the relation that holds true between the coordinates of every point on the curve, and no point that doesn't lie on the curve ii). To find the equation to a locus, we start by converting the given conditions to mathematical equations.

## Example 1

Find the locus of the point moving on a plane which is at a fixed distance 5 units from 'a' the $\mathbf{X}$ axis.

## Example 2

Find the locus of a point which is at a fixed distance 4 from the origin

## Example 3

of points that will satisfy more than one condition.

## Locus Theorems

## Locus Theorem 1: The

locus of points at a fixed distance, $d$, from point $P$ is a circle with the given point $P$ as its center and $d$ as its radius.

Locus Theorem 2: The locus of points at a fixed distance, $d$, from a line, $l$, is a pair of parallel lines d distance from I and on either side of $I$.

Locus Theorem 3: The locus of points equidistant from two points is the perpendicular bisector of the line segment determined by the two points.

## Locus Theorem 4: The

 locus of points equidistan from two parallel linesLocus Theorem 5: The locus of points equidistant from two intersecting lines

## Equation to a locus

"The equation of a curve is the relation which exists between the coordinates of all points on the curve, and

Find the locus of a point such that it is equidistant from two fixed points, $\mathbf{A}(\mathbf{1}, \mathbf{1})$ and $B(2,4)$
which does not hold for any point not on the curve".

Finding out the equation to a locus means finding out the relation that holds true between the $\mathbf{x}$ and $\mathbf{y}$ coordinates of all points on the locus.
Describe the Cartesian coordinate system (x-and -$y$-axes).
Demonstrate on a graph board to plot points.

Explain the meaning of the variables on the straight line $y=m x+c$

Solve problems on the distance between two given points $\left(x_{1}, y_{2}\right)$ and $\left(x_{2}, y_{2}\right)$ using the formula
$\mathrm{d}=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
solve the gradient of the two points $\left(x_{1}, y_{2}\right)$ and $\left(x_{2}, y_{2}\right)$
using gradient $=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
solve a problem on division of line segment in the ratio $m: n$ at the points
( $x_{1}, y_{2}$ ) and ( $x_{2}, y_{2}$ ) use the relation $\left(\frac{m x_{2}+n x_{1}}{m+n}, \frac{m y_{2}+n y_{1}}{m+n}\right)$

Organize students in pairs or groups. Ask simple multiple choice question.
Give simple class work.
Example. $A(3,4)$ and $B(5,9)$ are two point on a straight. Compute:
a) the distance
b) the slope
c) mid-point

Example.
$A(3,4)$ and $B(5,9)$ are two point on a straight. Compute the equation of the line,

Conduct quizzes and examinations

- Acute angle between two intersecting lines


## Co-ordinate Geometry

## The Circles

The equation of a circle with a given centre and radius
$(x-a)^{2}+(y-b)^{2}=r^{2}$ where ( $a, b$ ) is the centre and $r$ the radius.

Equation of circle through the ends of a diameter

Equation of a tangent and normal to a circle

## Conics Section -

## Parabola

Standard equation of a parabola

The equation of a parabola given the coordinates of the focus and equation of the directrix
solve problems on acute angle between lines

Describe a circle and state the general equation of a circle.

Teacher solve problems on finding the radius
$r=\sqrt{g^{2}+f^{2}-c}$ and the
center $(-g,-f)$ from the
general equation

$$
\begin{gathered}
x^{2}+y^{2}+2 g x+2 f y+c \\
=0
\end{gathered}
$$

Solve problems on tangents and normal to a curve

Explain the meaning of a parabola and discusses the shape of the curve when different conditions are given conditions

Teacher demonstrates how to sketch a parabola on a graph board

Solve problems on tangents and normal to a parabola

Arrange pupils in groups and give them tasks to do.

## Example.

1). Find the center and radius of the circle
$x^{2}+y^{2}-3 x+4 y=8$.
2). Sketch the circle whose general equation is
$2 x^{2}+2 y^{2}-3 x+16 y=8$.
3 ). Find the equation of the tangent to the circle $x^{2}+$ $y^{2}-2 x+4 y-1=0$.

Conduct quizzes and tests

Ask simple question about parabolas and record their responses on the board. Organize in groups and give tasks to do in class.

## Example 1.

Find the standard form of the equation of the parabola with vertex $(2,3)$ and focus $(1,2)$.

Example 2.
Find the equation of the tangent line to the parabola $y=x^{2}$ at the point $(1,-1)$

- the axis of symmetry


## Introduction to

trigonometry
Trigonometric ratios from the right triangle and corresponding reciprocals

Trigonometric ratios of known values
$\left(0^{0}, 30^{0}, 45^{0}\right.$, etc $)$
Basic relationships and trigonometric identities

Convert degrees to radians and vice versa

The quadrants and the sign of the trigonometric ratios

Applications of
trigonometry (solution of a triangle, elevation and depression) simple cases only.

## Trigonometric

equations and graphs
Solving trigonometric
equations
Trigonometric graphs

## Students should be able

to:

- Find sine, cosine and tangent of angles $0^{\circ} \leq \Theta \leq 360^{\circ}$ in general and $0^{\circ}$ $30^{\circ}, 45^{\circ}, 60^{\circ}$ and $90^{\circ}$ in particular
- Use the basic trigonometric ratios and reciprocals to prove given trigonometric identities
- Evaluate the sine, cosine and tangent of negative angles
- Convert degrees into radians and vice versa
- Apply trigonometric ratios and rules to real-life situations


## Students should be able

 to:- Use trigonometric identities to solve equations.
- Draw graphs of sine, cosine and tangent ratios in degrees and

Use the right triangle to derive the three basic trigonometric ratios and their corresponding reciprocals

Explain the use of the right triangle to give the
relationships between the
trigonometric ratios. $\tan x=$ $\frac{\sin x}{\cos x}$,
$\sin x=\cos (90-x)$
$\sec x=\frac{1}{\cos x}$ etc.
Explain the use of the quadrants for the sign of each trigonometric ratio

Solve simple problems on elevation and depression.

Explain and solve trigonometric equations

Draw graphs of the three basic trigonometric ratios and explain their nature and use the graphs to solve trigonometric equations

Ask students to name the basic trigonometric ratios. Arrange in groups and give the simple task to do whils you move around helping struggling students.

Example.
Evaluate $\cos 225^{\circ}, \sin 300^{\circ}$
Example.
Convert $330^{\circ}$ into radian
Convert $4 \pi$ into degrees

Give class work. Ask a pupil to come to the board and solve a given exercise.
Example.
Solve the equation $2 \sin x-$ $3 \cos x=1$ for $0 \leq x \leq 180$

Example.
Sole the equation $\sin 2 x=$ $\cos 5 x$
radians and
recognize their
periodic nature over
an extended
domain

- Use graphs to solve
trigonometric
functions up to quadratics, within a specified domain
- Calculate the
maximum and minimum points of given trigonometric functions

Example draw the graph of

$$
y=\sin x \text { for } 0 \leq x \leq 2 \pi
$$

## YEAR 2/TERM 3

## Limits

Definition of Limit of a function

Limit properties

1. Limits of constant
2. Limits of the function $x^{k}$
3. Limits of the function x
4. Limits of the function kx
5. Limits of the function $f(x) . g(x)$
6. Limits of rational functions
7. Limits involving infinity

## Students should be able:

- Define the concept of limits of a function.
- Apply the limit property to evaluate given functions
i). If $\lim _{x \rightarrow a} f(x)=k$ where k is
a constant, then $\lim _{x \rightarrow a} k=k$
ii). $\lim _{x \rightarrow a} x^{k}=a^{k}$
iii). $\lim _{x \rightarrow a} x$
iii). $\lim _{x \rightarrow a} x=a$
iv). $\lim _{x \rightarrow a} k x=k a$
v). $\lim _{x \rightarrow a} f(x) . g(x)=$
$\lim _{x \rightarrow a} f(x) \cdot \lim _{x \rightarrow a} g(x)$
$f(a) . g(a)$

Teacher to explain the concept of limits

Discuss with the students the properties or theorem of limits with given examples Example: Find $\lim _{x \rightarrow 2}(x+$
3) $\left(x^{2}-5\right)$

Solve problems with the students involving application of limit properties

## White board

1. $\lim _{x \rightarrow 2} x^{3}=2^{3}$
2. $\lim _{x \rightarrow 2} x=2$
3. $\lim _{x \rightarrow 5} 3 x=3(5)$
4. $\lim _{x \rightarrow 2}\left(x^{2}-4 x+2\right)$
5. $\lim _{x \rightarrow 2}\left\{\frac{x^{2}-7 x+10}{x^{2}-4}\right\}$
6. $\lim _{x \rightarrow \infty}\left\{\frac{\left.5 x^{2}-1\right)}{2 x^{2}+1}\right\}$
vi). $f(x)=\frac{g(x)}{h(x)}$, then
$\lim _{x \rightarrow a} f(x)=\frac{\lim _{\substack{x \rightarrow a \\ \lim _{x \rightarrow a} h(x) \\ x \rightarrow a}}=\frac{g(a)}{h(a)}, ~(a)}{}$ vii). $\lim _{n \rightarrow \infty} f(x)$.

## Students should be able

to:

- Define the derivative of a function
- Find the derivative of simple function. from pupit ons responses sh the board.
Gradient $=\frac{\text { increase } y}{\text { increase } x}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
Teacher explains that small increments were added to both x and y then $\frac{\Delta y}{\Delta x}=$ $\frac{f(x+\Delta x)-f(x)}{\Delta x}$.

Write the notations of differentiation $\frac{d y}{d x}$ or $f^{1}(x)$ all denoting first differentials

Solve problems with the students involving derivative of a function.
Teacher explains the method of finding derivative of function by first principles.

Teacher discuss with students how to differentiate common functions such as : $y=c, y=x^{n}$, etc

Electronics graph board Graph boards
Rulers
Graph papers
Give class work. Eg. Differentiate from first principles the function $y=$ $x^{2}$. Ask pupils to explain how they arrive at the answer

Group pupils and give them class activities on the concepts taught.
Eg. Use the quotient rule to find $\frac{d y}{d x}$ and $\frac{d^{2} y}{d x^{2}}$ for $y=\frac{2 x}{x+5}$.

Quotient rule
differentiation
Chain rule (also known as function of a function)

Successive
differentiation (higher derivatives)

- Differentiate a product using product rule.
- Eg. If $y=u v$
- then $\frac{d y}{d x}=u \frac{d v}{d x}+$
$v \frac{d u}{d x}$
- Eg. If $\mathrm{y}=\frac{u}{v}$

$$
\begin{aligned}
& \text { then } \frac{d y}{d x} \\
& =\frac{v \frac{d u}{d x}-u \frac{d v}{d x}}{v^{2}}
\end{aligned}
$$

- Differentiate a function of a function.

$$
\frac{d y}{d x}=\frac{d y}{d u} \times \frac{d u}{d x}
$$

- Differentiate a function successively. Eg. $\frac{d^{2} y}{d x^{2}}$


## Implicit Differentiation

How to differentiate
function of another function

Student should be able to:

- Use the chain rule to differentiate implicitly
- Find the slope of a curve at a given point.
- Apply the concept of implicit differentiation to find the equation of

Teacher can further discuss with pupils through questioning the meanings of product and quotient of numbers.

Apply the product and quotient rule to Differentiate functions
Eg. If $y=(2 x-2)\left(2 x^{3}\right)$
(Product rule)
Eg. If $\mathrm{y}=\frac{(2 x-2)}{\left(2 x^{3}\right)}$
(Quotient rule)
Solve problems on
Differentiating function of a function.

Teacher to introduce higher
or successive
differentiation.

Explain the meaning of
implicit functions.
Eg $x^{2}-3 x y^{2}-y=6$
Explain to pupils how to differentiate implicitly

Solve problems on implicit Differentiating as work examples

## a tangent to a curve

at a given point

## Derivative of Trig

 FunctionsHow to determine the derivative of a trigonometric function with a given function.

Differentiation of natura log functions and exponential functions

## Applications of

## differentiation

Increasing and decreasing functions Rates of change,
Velocity and acceleration, Turning points (maximum and minimum)
Points of inflexion
Tangents and normal
Practical problems

## Student should be able to

- Compute the differentials of trigonometric functions
- Apply the echniques of differentiation to calculate the differentials of trigonometric functions
- Differentiate composite trigonometric functions.
- Differentiate logarithmic functions. Such as $y=\log _{e}(2 x-5)$

Students should be able to:

- Describe an increasing and decreasing function.
- Apply differentiation to Determine
I. rates of change
II. velocity and acceleration (maximum and minimum)

Discuss with pupils the three basic trigonometric ratios
$(\sin x \cdot \cos x$ and $\tan x$ ) with their corresponding
reciprocals
$(\csc x \cdot \sec x$ and $\cot x)$ using the right-triangle.

Solve problems on Differentiating trigonometric ratios applying the techniques of differentiation.

Solve problems on Differentiating logarithmic and exponential functions applying the techniques of differentiation

Teacher to discuss with the students meaning of rate of change, Velocity and acceleration, Turning points (maximum and minimum).
Explain that at a turning
point $\frac{d y}{d x}=0$.
Solve problems as work examples on some application of differentiation.

Ask pupils to list the trigonometric ratios. Record their responses on the board.

Ask pupils to find the differential coefficient of $y=$ $\sin x$. Ask one or two pupils to try and solve it on the board.

Ask pupils to explain velocity and acceleration.

Give pupils some class work for them to try.
Find the maxima and minima points of the functiony $=(2 x-1)(4-$
$x)^{2}$.
IV. Tangents and normal

## Integration

Process of Integration
The general solution of An Indefinite integral and a Definite integral

## Techniques of

## integration

Introduction to
integration of
Trigonometric Functions.
Integration by
substitution
Integration of Logarithmic functions

Integration of exponential functions.
Some applications of
integration
Area under curves

## Practical problems

## Students should be able

to:

- Define integration as the reverse of differentiation
- Determine the integrals of the form $x^{n}$ and $a x^{n}$. Where n is a fractional, zero, or positive or negative integer.
$\int x^{n} d x=\frac{x^{n+1}}{n+1}+c$
(indefinite integral)
$[x]_{a}^{b}=(b)-(a)$ (definite integral)

Student should be able to:

- Integrate simple trigonometric functions $\int \sin x d x$.
- Integrate functions by substitution method
- Integrate logarithmic functions $\left(\int \ln x d x\right)$
- Integrate exponential functions ( $\int e^{x} d x$ )

Students should be able to: Students should be able to:

Explain to pupils the meaning of integration and he notation for integration as $\int$
Solve problems on indefinite integrals
$\int x^{n} d x=\frac{x^{n+1}}{n+1}+c . \mathrm{C}$ is the arbitrary constant also known as the constant of integration.
Explain the concept of definite integral $[x]_{a}^{b}=(b)-$ (a).

Solve some mathematical problems on the definite and indefinite integrals.

Ask pupils to state the basic trigonometric ratios.
Explain and guide pupils to integrate trigonometric functions.
Discuss with pupils the process of substitution in integration.
Explain how to integrate logarithmic and exponential functions.

Discuss the concept of definite integral to find the area $\left(\int_{a}^{b} f(x) d x\right.$ or $\left.\int_{a}^{b} y d x\right)$

Ask pupils to give the difference between differentiation and integration

Give pupils (groups) exercises to try in class. Eg. integrate $x^{2}$
Eg find $\int_{1}^{2}(3 x-4) d x$

Integrate $\sin x$ and $\cos x$.
Eg. Find $\int \frac{1}{2 x} d x$.

Give class work to pupils whilst you walk around supervising.

- Apply integration to calculate areas under curves
- Apply the trapezoidal rule to evaluate the area under a curve
and the volume of a solid obtained by rotating the area bounded by the curve $\left(V=\pi \int_{a}^{b}(f(x))^{2} d x\right)$
Explain the use of
trapezium rule.
Solve problems on the
applications.


## Vectors

Vectors and scalars Properties of vectors (representing vectors, equal vectors, null or zero vector)

The magnitude and direction of a vector

Algebra of vectors
Triangle law of vector addition

## Students should be able

 to:- Describe vector and scalar quantities
- Write the notations for a vector and represent a vector on the rectangular Cartesian coordinate system.
- Compute the magnitude and direction a vector
- Apply the algebra of vectors including:( addition, subtraction and scalar multiplication of vectors)
- Use the geometric applications of vectors on the triangle, the parallelogram and other polygons using the laws of


## YEAR 3/TERM 1

Explain vectors and scalars quantities with given
examples to each
Demonstrate the representation of vectors on a Cartesian plane using the graph

Discusses the various ways of notating a vector. Eg $\overrightarrow{A B}$ (directed line segment joining two points from A to
B) or as components of a point that is $\binom{x}{y}$. Bold type letter is another way of notating a vector.

Calculate the magnitude as
$|\overrightarrow{A B}|=\sqrt{X^{2}+Y^{2}}$ and the
direction as $\theta=\tan ^{-1}\left(\frac{Y}{X}\right)$
Discuss the geometric approach to solve vector

Eg. Find the area bounded by the curve $y=4 x^{2}$, the $x$ axis and the ordinates $x=0$ and $x=1$

Ask students to give examples of vector and scalar quantities. Record all responses on the board

Ask them to represent a vector on the board

Give them group work.
Example.
A girl walks $x k m$ due east
then zkm north-east.
Calculate the total distance she has walked and her displacement from her starting point when $x=$ 3 and $z=4$
addition and subtraction of vectors

## Kinematics of a

## particle

Motion in a straight line with constant acceleration
Vertical motion under gravity
Speed- time graphs

## Statics

Resultant and Resolving forces into components

Equilibrium of coplanar forces

Students should be able to:

- Define kinematics and other related terminologies and state their unit of measurement.
- Derive the equations of linear motion with uniform acceleration
- Solve problems on acceleration due to gravity
- Solve uniform accelerated motion problems graphically


## Students should be able

 to:- Explain the meaning of statics
- Resolve forces and calculate the resultant force
problems using the triangle law of vector addition

Explain terminologies on uniform motion
(displacement, velocity, acceleration, distance,

## speed)

Apply the definitions of the terminologies to derive the equations of uniformly accelerated motion. That is

$$
a=\frac{v-u}{t}
$$

$v^{2}=u^{2}+2 a s$.
$s=\left(\frac{u+v}{2}\right) t$.
$s=u t+\frac{1}{2} a t^{2}$.
Solve problems on uniform motion graphically

Apply the concept of uniformly accelerated motion to solve problems on vertical motion.
Discuss the meaning of statics.

Explain the resultant of forces and help students to
resolve a force into
components forces and compute the resultant force.

Ask students to define speed, velocity, distance, displacement and acceleration. Record their answers on the board.

Group them and give work to do in class.

Example.
A particle is moving in a straight line with uniform acceleration. If it travels 120 m while increasing speed from
$5 \mathrm{~ms}^{-1}$ to $25 \mathrm{~ms}^{-1}$ find its acceleration.

Conduct quizzes and tests

Organize them in group and give them class exercises

Example.
A force $F$ acts on a particle
at an angle of $\theta$ to the horizontal. Find the horizontal and vertical

Types of forces ( weight tension and trust)

Friction and coefficient of friction

## Dynamics

Rotational motion of rigid
bodies (moment of inertia of a particle and rigid body)

Newton's laws of motion
Motion of two connected particles (problems
involving pulleys)
Momentum and impulse ( principle of
conservation of
momentum)

- Solve problems on the equilibrium of coplanar forces
- Explain friction and resolve a contact force into normal and frictional components


## Students should be able

 to:- Define a rigid body
- State and explain Newton's laws of motion
- Solve problems using Newton's laws of motion
- Explain the meanings of momentum and impulse and how they are related
- Solve problems on conservation of linear momentum
$R=\sqrt{X^{2}+Y^{2}}$,
where $\mathrm{X}=$ horizontal
component $\quad \mathrm{Y}=$
vertical component
Explain coplanar forces and solve some problems

Discuss friction and demonstrate the resolution
of the normal and friction components

Use the relation $F=\mu R$ to solve friction related problems
Define and explain rigid
body
Explain that moment of inertia of rigid body = sum of moments of inertia all the particles present in the
body, ie
$I=m_{1} r^{2}{ }_{1}+m_{2} r^{2}{ }_{2}++\cdots$. $\rightarrow I=\sum m r^{2}$.

Discuss Newton's laws of motion with practical examples

Establish the relationship between impulse and momentum.
That is impulse $=$ change
momentum, $I=m(v-u)$
Explain the principle of
conservation of momentum.
components of F when $\mathrm{F}=$ 20 N and $\theta=20^{\circ}$.

Ask students to explain the types of forces.

Ask students to state and explain the laws of motion

Organize students in groups and administer task to do.
Example.
Find the resultant force which will produce an acceleration of $5 \mathrm{~ms}^{-2}$ for a particle of 6 kg .

Example
A car of mass 800 kg
decelerates from $20 \mathrm{~ms}^{-1}$ to $5 \mathrm{~ms}^{-1}$. Find the loss of momentum.

## Moments

Sum of moments

Equilibrium of a lamina under parallel forces (non-uniform rods)

## Students should be able

 to:Explain moments of a force
Solve problems on uniform and non-uniform rods

That is
Total momentum before
impact $=$ total momentum
after impact or $m_{1} u_{1}+$
$m_{2} u_{2}=m_{1} v_{1}+m_{2} v_{2}$
Explain moment of force.
Describe non-uniform and uniform rods.
Solve problems

## YEAR 3/TERM 2

## Matrices

Operations on matrices
Finding the determinant and inverse of a matrix (limited to $2 \times 2$ matrices)

Application of matrices (Cramer's rule) to solve simultaneous linear equations in two variables

## Students should be able

to:

- Define matrix, the order of matrix (ie. $2 \times 2,2 \times 3$ etc) and recognize the types of matrices
- Explain the operations (addition, subtraction and multiplication of matrices up to $3 \times 3$ order) and solve problems

Students are organized in groups and given tasks to do.

## Example.

A uniform rod $A B$ of length 5 m and mass 6 kg is pivoted at $C$ where $A C=1.5 \mathrm{~m}$.
Calculate the mass of the particle which must be attached at $A$ to maintain equilibrium with the rod horizontal.

Ask pupils to explain different orders of matrices.

Organize students in groups and give the class exercises to solve.

|  | Explain the determinant and its solution <br> Solve problems on the inverse of a $2 \times 2$ matrix. | $A^{-1}=\frac{1}{a d-b c}\left(\begin{array}{cc} d & -b \\ c & a \end{array}\right)$ |
| :---: | :---: | :---: |
| Linear <br> Transformations <br> The concept of linear transformation | Student should be able to: <br> - Define transformation and explain image and object | With cited examples ask students about the common meaning of transformation <br> Explain with illustrations in |
| Images of points under given linear transformation | - Fine the images of points and objects under linear transformation | finding the inverse of a linear transformation <br> Solve problems on the use |
| Determine the matrices of linear transformation | - Find the matrix and inverse matrix of a | of identity transformations. Examples: <br> $\left(\begin{array}{ll}1 & 0 \\ 0 & -1\end{array}\right)$ |
| The inverse of linear transformation | transformation. <br> - Find composition of | $\left(\begin{array}{ll}1 & 0 \\ 0 & -1\end{array}\right)$ reflection on the $x$ - axis |
| Composition of linear transformation | linear transformations. <br> Such as $H=F o G=$ $F G=F(G)$ (means take transformation $G$ and then apply transformation F to it in that order\} <br> - Recognize the identity transformations <br> - Find the equation of the image of a line under a given linear transformation | $\left(\begin{array}{cc}-1 & 0 \\ 0 & 1\end{array}\right)$ reflection on the $y-$ axis $\left(\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right)$ reflection in the line $y=x$ $y=x$ |

## YEAR 3/TERM 3

## Statistics - Data Representation

Grouped Data

## Statistics - Data

Analysis
Measures of Central
Tendency (Grouped
Data)

## Students should be able

to:

- Define statistical terms
- Represent statistical data using Frequency distribution tables, Histograms, Cumulative frequency curve


## Students should be able

 to:- Calculate mean, mode, median, quartiles, and percentiles
- Estimate mode and modal class for

Review the concept of ungrouped data. Introduce the topic by defining statistics as way of collecting, ordering, analyzing and interpreting data for proper decision making.

Explain some statistical terms:
Discrete data, continuous data, frequency, frequency distribution table, class interval

Discuss how data are represented by:
a). Frequency distribution
tables
b). Histograms
c). Cumulative frequency
illustrate an example on the graph board how to construct histogram and the ogive curve
Explain to the students meaning of mode, mean, median, quartiles, and percentiles

Discuss with the students terminologies and columns used as formula to calculate

Graph board
Graph paper
Blackboard ruler
Foot rule
Markers
Colored chalks
Pencils
Electronic Graph board
Graph paper
Blackboard ruler
Foot rule
Markers
Colored chalks
Pencils

Write a short essay on the origin and development of the science of Statistics.
(b) Discuss the utility of Statistics to the state, the economist, the industrialist.. in a planned economy.

Example
Draw histogram for the following frequency distribution.
Variable : 10-20, 20-30,30-
40, 40-50, 50-60,60-70,
70-80
Frequency : 123035654525 18 respectively

Give students class exercises on measures of central tendencies for grouped and ungrouped data.

Example:
grouped data from a histogram

- Estimate median and mean from grouped data a histogram
- Calculate mean and standard deviation, variance, range, inter-quartile range

Measures of Dispersion (Grouped Data)

## Statistics - Correlation

 and Regression
## Mode for grouped data

Mode $=L+\left[\frac{\Delta_{1}}{\Delta_{1}+\Delta_{2}}\right] C$
Median for grouped data
Median
$=L+\left[\frac{\frac{1}{2} N-\left(\sum f\right) L}{f_{\text {median }}}\right] C$
Mean for grouped data
$\bar{x}=\frac{\sum f x}{\sum f}$
Demonstrate how to
estimate mode, median and mean for grouped data using histogram.

Solve work examples using the various formulae above.

Demonstrate how to Calculate mean and standard deviation, variance, range, interquartile range.
Explain correlation as means of determine the relationship between two variables.
Further discuss with the students that the two variables are independent

Demonstrate using the chart to describe the forms of correlation.

Chart showing types of correlation.

Chart -- Line of best fit. 2 Chart - illustrating scatter diagram

Electronic Graph board Graph paper Blackboard ruler
Foot rule

Construct a grouped frequency table and use the table to calculate the mean median and mode for the data below
10, 20, 22, 67, 45, 43, 20
14, 34, 54, 76, 43, 32, 21
22, 12, 23, 34, 54, 67, 77,
$56,66,54,43,76,66,54$
34, 32, 23, 43, 23, 25, 32
12, 21, 23, 35, 34

Compute the Spearman rank correlation

| History | Algebra |
| :--- | :--- |
| 35 | 30 |
| 23 | 33 |
| 47 | 45 |
| 17 | 23 |
| 10 | 8 |
| 43 | 49 |
| 9 | 12 |
| 6 | 4 |
| 28 | 31 |

method

- Draw the line and finding the equation of best fit (Regression)

Probability Introductory concepts

Students should be able to:

- Define probability and related terminologies (Sample space, Outcome,
Observation, Events, relative frequency, occurrence, not occurrence and experiment)
- Describe probability events: Equally likely events,
Mutually exclusive


## (positive correlation,

 negative correlation and no correlation)Illustrate scatter diagram on a graph board with pair values of two variables.

Solve problem with the students to calculate the correlation coefficient using spearman's rank method.
*Use data without ties.

$$
r=1-\frac{6 \sum d^{2}}{n\left(n^{2}-1\right)}
$$

$-1 \leq r \leq 1$
Explain the concept of regression line and the equation line of best fit. Introduce probability as concept of chance and occurrence of event.

Discuss the related terminologies with cited examples.
Sample space, Outcome, Observation, Events, relative frequency, occurrence, not occurrence and experiment.

Describe probability events a). Equally likely events

Markers
Colored chalks
Pencils

Fair coin
Fair dice
Colored balls
Playing cards Colored marbles

Random experiments sample spaces, and events

A ball is drawn at random from a box containing 6 red balls, 4 white balls, and 5 blue balls. Determine the probability that it is (a) red, (b) white, (c) blue, (d) not red, (e) red or white.

## independent events

A fair die is tossed twice.
Find the probability of
getting a 4,5 , or 6 on the first toss and a 1, 2, 3, or 4 on the second toss.
events,

Independent events

- Calculate
probability value of experiment (coin flip and dice toss)
- Apply addition rule or product rule of probability
- Identify conditional probability
- Draw objects with and without replacement
- Construct the tree diagram

Probability Permutations and Combinations
$\begin{aligned} P(E) & =\frac{n(E)}{n(S)}\end{aligned}$
For $0 \leq p(E) \leq 1$
b). Mutually exclusive events
If $A$ and $B$ are mutually exclusive then.

$$
P(A \cup B)=P(A)+P(B)
$$

## c). Independent events

If $A$ and $B$ are mutually exclusive then.
$P(A$ and $B)=P(A) \times P(B)$
Calculate probability value of various events (coin flip and dice toss etc ). Apply addition rule or product rule of probability

Draw objects with and without replacement

Discuss conditional probability and help students to Construct the tree diagram.
Introduce the fundamental counting principle

Explain the factorial notation (ie. $n$ ! Read as $n$ factorial)

Discuss what permutation and combinations are:

One bag contains 4 white balls and 2 black balls; another contains 3 white balls and 5 black balls. If one ball is drawn from each bag, find the probability that (a) both are white, (b) both are black, (c) one is white and one is black.

Chart illustrating permutation and combinations formula

Arrange students in groups and give tasks to do.

Example:
Evaluate the value of 7 !
Find the permutation and combination
if $n=12$ and $r=2$.

- Count outcome of events using combination (order of choices not considered)

Probability - Binomial Distribution

Permutation as order of choices considered
$P_{r}^{n}=\frac{n!}{(n-r)!}$
$=n(n-1)(n-2)(n-$
3) $\ldots(n-r+1)$

Combinations order of choices not considered.
$C_{r}^{n}=\frac{n!}{(n-r)!r!}$

Solve problem on permutation and combination.

Teacher to recap the concept of combination.

Describe what binomial distribution as any situation having only two possible mutually exclusive outcomes.
Eg success and failure...
boy or girl.
Explain that the binomial distribution probability function can be expressing
as:
$\boldsymbol{P}(X=x)=C_{r}^{n} \boldsymbol{p}^{x} \boldsymbol{q}^{\boldsymbol{n - x}}$
If $p$ is the probability that the event will happen and the probability of not happen is $q=1-p$

In how many ways of 4 girls and 7 boys, can be chosen out of 10 girls and 12 boys to make the team?

How many words can be formed by 3 vowels and 6 consonants taken from 5 vowels and 10 consonants?

Suppose 30 people are in a room. What is the probability that there is at least one shared birthday among these 30 people?
A coin is tossed 10 times. What is the probability of getting exactly 6 heads?
$80 \%$ of people who purchase pet insurance are women. If 9 pet insurance owners are randomly selected.
Find the probability that exactly 6 are women.

60\% of people who purchase sports cars are men. If 10 sports car owners are randomly selected.
Find the probability that exactly 7 are men.

## Resources

- Vanguard
- Graph Board
- Graph paper
- Blackboard ruler
- Foot rule
- Pencil
- Permanent Markers
- Graphing software/laptop
- Coin
- Dice
- Playing Cards
- Coloured Marbles
- Coloured Counters
- A3 Plain paper

